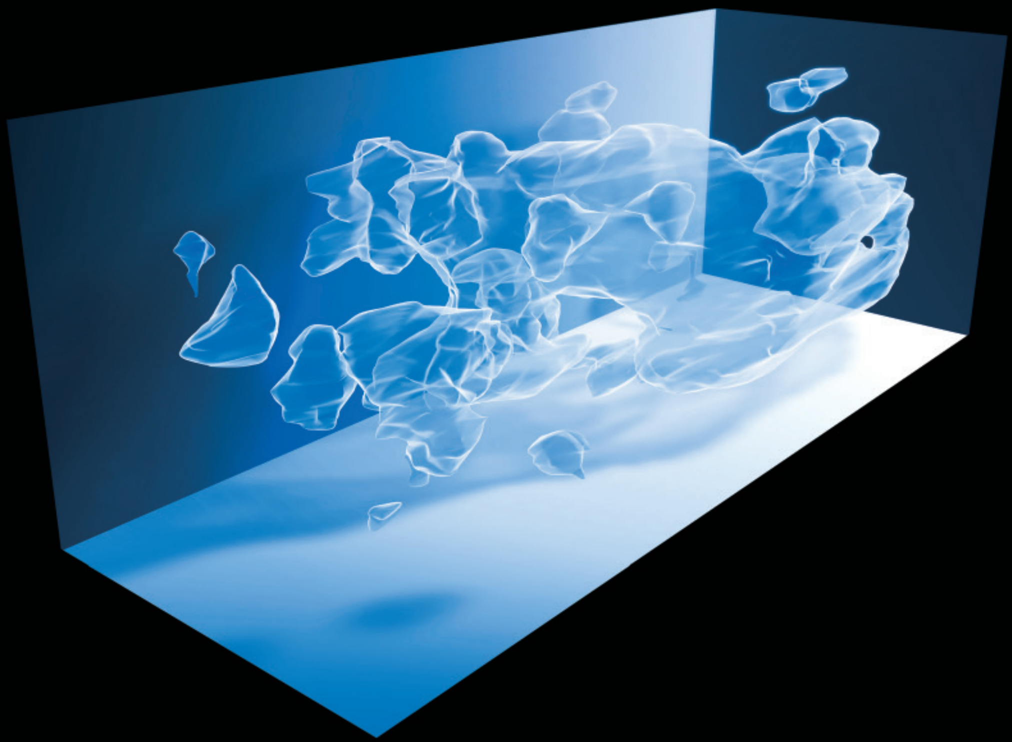
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



01 Perimeter Explorations

The Mystery of Dark Matter

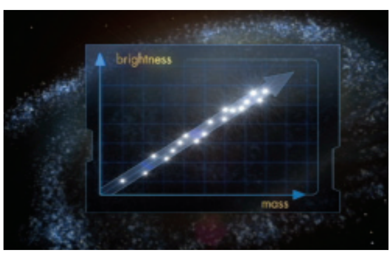
TEACHER'S GUIDE

Dark Matter
in a Nutshell

- In 1967, Vera Rubin observed that stars within the Andromeda galaxy had higher-than-expected orbital speeds.
- Physicists have also observed the same phenomenon in the nearby Triangulum galaxy.
- By measuring the orbital speeds of stars within Triangulum and using the formula

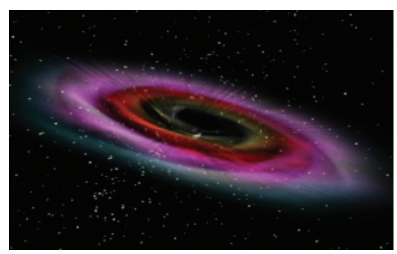
$$M_{\text{gal}} = \frac{v^2 r}{G}$$

  physicists have calculated that the mass of this galaxy within a radius of r = 4.0×10^{20} m is equivalent to 46 billion Suns.
- However, by measuring the brightness of Triangulum, they have also calculated that its mass within a radius of $r = 4.0 \times 10^{20}$ m is equivalent to 7 billion Suns.
- The discrepancy between these two results implies that there is 39 billion Suns of unseen mass within Triangulum.
- This unseen mass is called "dark matter."
- Physicists have observed many other galaxies and most are now convinced that, on average, dark matter accounts for 90% of the mass of every single galaxy in the universe.
- Physicists also have independent evidence for the existence of dark matter from observations of distorted images of distant galaxies (gravitational lensing).
- Although no one knows what dark matter is made of, physicists currently have a number of theories.
- One of the earliest theories of dark matter was that it consists entirely of compact celestial objects such as planets, dwarf stars, and blackholes. Careful observations have ruled out this theory.
- Most physicists today think that dark matter is made of a type of subatomic particle called a WIMP that, to date, has never been detected in the laboratory.
- Numerous experiments that are trying to detect one of these particles are currently underway worldwide.
- Dark matter is not related to dark energy. Dark energy refers to a mysterious mechanism pushing the universe apart.



Mass-Luminosity Relationship


The Andromeda galaxy


Black hole

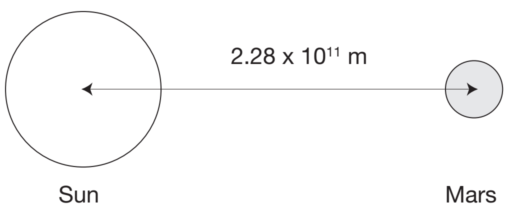
Activity 2
Key Concepts

NAME : ....

Useful formulas

$$F_{\text{net}} = ma \qquad F_{\text{C}} = \frac{mv^2}{r} \qquad F_{\text{G}} = \frac{GMm}{r^2} \qquad G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2 \qquad E_{\text{K}} = \tfrac{1}{2}mv^2$$

1. Mars orbits the Sun in uniform circular motion. The radius of Mars' orbit is 2.28×10^{11} m and its orbital speed is 2.41×10^{4} m/s.



a) Draw the free-body diagram for Mars and use it to derive an expression for the mass of the Sun in terms of Mars' orbital speed, the radius of its orbit, and the universal gravitational constant.

b) Use the expression derived in part a) to determine the mass of the Sun.

2. The plot below relates the orbital speed of the planets to the radius of their orbits.

a) What is the force that keeps the planets in their orbits?

b) Why do the "outer" planets travel slower than the "inner" planets?

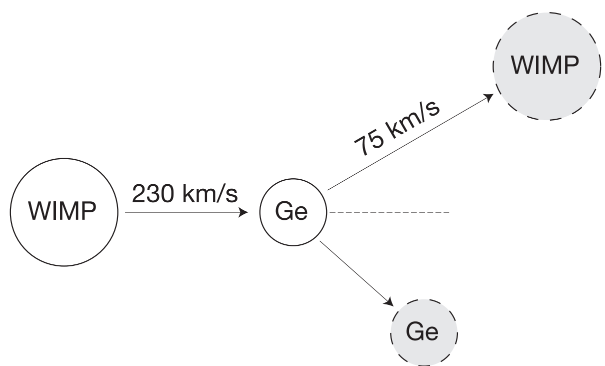
c) Rearrange your answer to 1a) to find the equation for the graph above.

3. Astronomers have studied galaxy UGC 128 for many years. They have measured its brightness and calculated that the mass of stars within a radius of 1.30×10^{21} m is 3.34×10^{40} kg. Stars orbiting at this radius has been measured travelling at a speed of 1.30×10^{5} m/s. What percentage of the mass within this radius is dark matter?

4. In rural Minnesota, U.S.A., there is a dark matter detector known as the Cryogenic Dark Matter Search (CDMS) located 700 m underground in an abandoned mine. It involves a number of 250 g crystals of germanium (Ge) that are cooled down to just above absolute zero (–273 °C).

   According to the weakly interacting massive particle (WIMP) theory of dark matter, billions of WIMPs from outer space are raining down on Earth each second. Although they typically pass through solid objects as if they are not there, there is a very small chance that a WIMP will collide with a nucleus of an atom within any material it happens to pass through.

   As a result, at CDMS there is a very small probability that a WIMP will collide with the nucleus of a germanium atom within the detector, as illustrated below:



a) In the figure above, a WIMP with a mass of 1.07×10^{-25} kg and an initial speed of 230 km/s collides with a stationary germanium nucleus with a mass of 1.19×10^{-25} kg. If the WIMP is deflected and its speed is reduced to 75 km/s, use conservation of energy to determine how much energy is transferred to the nucleus. (It is this energy that scientists must somehow detect.) In calculating your answer, assume that the collision is elastic.

b) How many times smaller is this energy than the energy required to lift a grain of sand by one millimetre (1×10^{-7} J)?

5. A friend sends you an email that expresses skepticism about the existence of dark matter. It says: "I thought science was about observation, and objects you can see? How can you say that dark matter exists when no one can see it?" Write a five to ten sentence reply describing the evidence for dark matter and defending the stance that something does not have to be visible in order to be understood by science. In your reply, give an example from everyday life of something that exists but is not visible.

Activity 4
Dark Matter Within a Galaxy

NAME :

Useful formulas

$$F_{\text{net}} = ma \qquad F_C = \frac{mv^2}{r} \qquad F_G = \frac{GMm}{r^2} \qquad G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$$

Orbital radius of star ($\times 10^{20}$ m)	Measured speed ($\times 10^5$ m/s)	Calculated speed ($\times 10^5$ m/s)	Gravitational mass ($\times 10^{41}$ kg)	Missing Mass (%)
1.85	2.47	2.36	1.69	8.99
2.75	2.40			
3.18	2.37			
4.26	2.25			
6.48	2.47			

Astronomers have analysed the stars in the galaxy UGC 11748. They found that most of the stars lie within a radius r = 1.64 × 10^{20} m and that the total mass within this radius is 1.54 × 10^{41} kg, or 77.4 billion times the mass of the Sun.

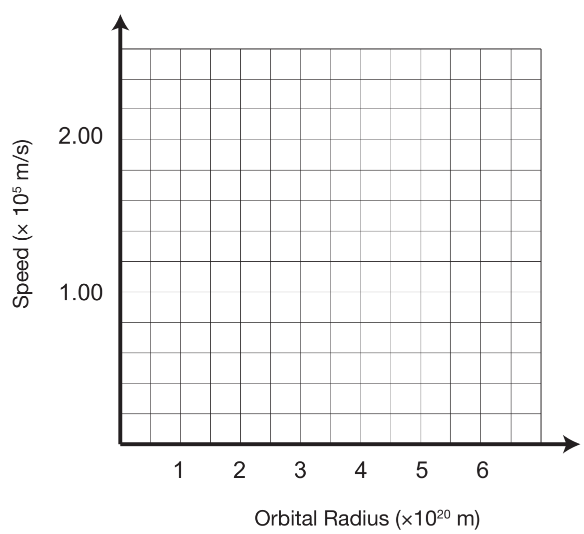
It is expected that the stars that lie outside this radius will orbit in the same way that planets orbit the Sun. In this activity you will analyse the motion of stars located in the outer regions of UGC 11748.

1. Use the values from the table above to plot measured speed against orbital radius on the graph provided. Label this line "measured".

2. a) For each orbital radius, calculate the speed expected if the only mass is the luminous mass of 1.54 × 10^{41} kg. Record your answers in the "Calculated speed" column.

   b) Show a sample calculation.

   c) Plot calculated speed against orbital radius on the graph provided. Label the line "calculated".



3. Compare the "measured" and "calculated" plots. Discuss a possible explanation for any differences.

4. a) Use the measured speeds to calculate the mass of the galaxy contained within each orbital radius. Record your answers in the “Gravitational mass” column.

   b) Show a sample calculation.

5. For each orbital radius, calculate the difference between the gravitational mass within this radius and the total mass of the visible stars (1.54×10^{41} kg). Represent this difference as a percentage of the gravitational mass within the orbital radius. Record your answers in the “Missing Mass” column.

6. Do your results support the following statement? “It is reasonable to expect that stars orbit around the gravitational mass contained within the radius of their orbit in the same way that planets orbit around the Sun.” Discuss.

7. **Explain** the shape of your plot for measured speed against orbital radius.