Heisenberg Uncertainty and Single Slit Diffraction

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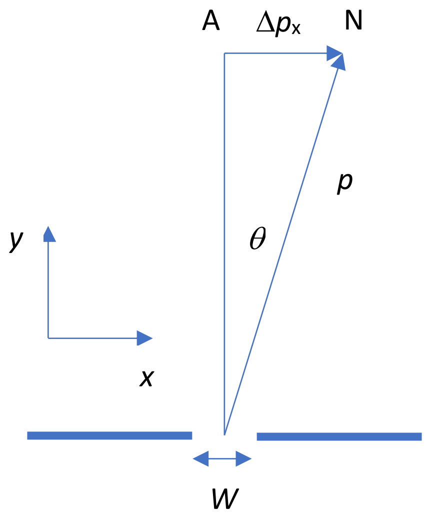
Consider a light beam (wavelength = λ) traveling upward through a narrow slit of width *w.*



For single slit diffraction of light waves at a distance (d) from the slit that is large compared to the λ and w, *i.e.* $d >> \lambda, w$, we can use the single slit diffraction equation that relates the angle (θ), between the central antinode (A) and the first node (N), to the wavelength (λ) and the slit width (w):

$$\sin\theta = \frac{\lambda}{w} \quad . \tag{1}$$

Now consider the light beam as consisting of a stream of photons traveling upward in the y-direction. As a consequence of going through the narrow slit, some of the photons acquire an x-momentum component that has a maximum value of Δp_x for photons in the central region of the diffraction pattern. The uncertainty in location of the photons as they travel through the slit is $\Delta x = w$.

Recall the energy and momentum relations for photons:

$$E = h\nu = \frac{hc}{\lambda} \quad \text{and} \quad E = pc \quad \text{yielding} \quad p = \frac{h}{\lambda} \quad . \tag{2}$$

Combining (1) and (2) with the momentum triangle in the diagram gives us

$$\sin(\theta) = \frac{\Delta p_x}{p} = \frac{\Delta p_x}{(h/\lambda)} = \frac{\lambda}{w} = \frac{\lambda}{\Delta x} \quad .$$

Rearranging yields $$(\Delta p_x)(\Delta x) = h \quad . \tag{3}$$

This is the relation followed by the beam of C-70 fullerene molecules in the QuarkNet "What Heisenberg Knew" Data Activity.