Heisenberg Uncertainty and Particle Energy

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Inspired by Ken Cecire's derivation of the connection between the Heisenberg relations

$$(\Delta p)(\Delta x) \geq \hbar/2 \qquad \text{and} \qquad (\Delta E)(\Delta t) \geq \hbar/2 \ ,$$

I offer the following. This derivation incorporates the relativistic relations for a particle of invariant mass *m*, speed *v*, momentum *p*, and total energy *E*:

$$E = \gamma m\ , \qquad p = \gamma m v\ , \qquad E^2 = m^2 + p^2\ , \qquad \text{where } \gamma = \frac{1}{\sqrt{(1-v^2)}}$$

and units are chosen so that $c = 1$.

$$(\Delta p)(\Delta x) = (\Delta p)(\Delta x)\left(\frac{\Delta t}{\Delta t}\right) = (\Delta p)\left(\frac{\Delta x}{\Delta t}\right)(\Delta t) = (\Delta p)v(\Delta t) \qquad (1)$$

$$p^2 = E^2 - m^2$$

Differentiating both sides yields $\quad 2p(dp) = 2E(dE)$,

or, for small finite differences, $\quad (\Delta p) = \frac{E}{p}(\Delta E) = \frac{\gamma m}{\gamma m v}(\Delta E) = \frac{1}{v}(\Delta E) \qquad (2)$

Inserting (2) into (1) yields $\quad (\Delta p)(\Delta x) = \frac{1}{v}(\Delta E)v(\Delta t) = (\Delta E)(\Delta t)$.

Thus, $\quad (\Delta p)(\Delta x) \geq \hbar/2 \quad$ implies $\quad (\Delta E)(\Delta t) \geq \hbar/2$. QED