Neutron Motion in a Target Nucleus

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One of the exercises offered by the MINERvA experiment group in their "Neutrinos in the Classroom" website (https://neutrino-classroom.org/) provides data on collisions between muon neutrinos and neutrons. The neutrons are contained in nuclei, most likely C-12, of hydrocarbon scintillator material in the MINERvA detector at Fermilab. Analysis of the data indicates that the neutrons have some motion in the *xy*-plane transverse to the neutrino beam, which travels in the *z*-direction. In the Neutrinos in the Classroom data set, the target neutron transverse momentum range is 39 to 3189 MeV/c. The detector is stationary.
Where does this neutron motion originate?

(1) Could the measured neutron momentum come from thermal vibrations of the classical (Maxwell-Boltzmann statistics) atoms in the scintillator?

Suppose the atoms in the scintillator are at temperature $T = 300$ K.
Kinetic theory for nonrelativistic atomic motion indicates

$$<KE_{\text{atom}}> = <p^2_{\text{atom}}>/2m_{\text{atom}} = (3/2)k_B T = 1.5(1.38 \times 10^{-23} \text{ J/K})(300\text{K})$$
$$= 6.21 \times 10^{-21} \text{ J} = 3.881 \times 10^{-8} \text{ MeV}.$$

Let us assume the neutrino collided with a neutron in the nucleus of a C-12 atom, and the nucleus vibrates in unison with the atom. We can calculate the atomic (and nuclear) motion as follows:

$$m_{\text{atom}} = 12 \text{ u} = 12\,(1.66 \times 10^{-27} \text{ kg}) = 1.99 \times 10^{-26} \text{ kg} = 1.12 \times 10^{4} \text{ MeV/c}^2.$$

$$<p^2_{\text{atom}}> = 2m_{\text{atom}}<KE_{\text{atom}}>$$
$$= 2(1.99 \times 10^{-26} \text{ kg})(6.21 \times 10^{-21} \text{ J}) = 2.47 \times 10^{-46} \text{ kg}^2\text{m}^2/\text{s}^2$$
$$= 2(1.12 \times 10^{4} \text{ MeV/c}^2)(3.881 \times 10^{-8} \text{ MeV}) = 8.69 \times 10^{-4} \text{ MeV}^2/\text{c}^2$$

$$p_{\text{atom-rms}} = \sqrt{< p^2_{atom} >} = m_{\text{atom}}v_{\text{atom-rms}} = 1.57 \times 10^{-23} \text{ kg m/s}$$
$$= 2.95 \times 10^{-2} \text{ MeV/c}$$

$v_{\text{atom-rms}} = p_{\text{atom-rms}}/m_{\text{atom}} = 789$ m/s, which is certainly a nonrelativistic speed.

The momentum of a neutron moving at $v_{\text{atom-rms}}$ is

$$p_{n789} = m_n\, v_{\text{atom-rms}} = (1.675 \times 10^{-27} \text{ kg})(789 \text{ m/s}) = 1.32 \times 10^{-24} \text{ kg m/s}$$
$$= 2.47 \times 10^{-3} \text{ MeV/c}.$$

This value is at least 4 orders of magnitude to small to account for the calculated transverse neutron momentum.

(2) Could the measured neutron momentum result from the neutrons and protons in a target nucleus being in classical (Maxwell-Boltzmann statistics) thermal equilibrium with the scintillator atoms?

Again assume that the equilibrium temperature is 300 K.
If we apply kinetic theory to the nucleons in a nucleus, we have

$$<KE_n> = <p^2_n>/2m_n = (3/2)k_B T = 1.5(1.38 \times 10^{-23} \text{ J/K})(300\text{K})$$
$$= 6.21 \times 10^{-21} \text{ J} = 3.881 \times 10^{-8} \text{ MeV}.$$

Then neutron momentum is given by

$$<p^2_n> = 2m_n<KE_n>$$
$$= 2(1.675 \times 10^{-27} \text{ kg})(6.21 \times 10^{-21} \text{ J}) = 2.08 \times 10^{-47} \text{ kg}^2\text{m}^2/\text{s}^2$$
$$= 2(9.396 \times 10^2 \text{ MeV/c}^2)(3.881 \times 10^{-8} \text{ MeV}) = 7.29 \times 10^{-5} \text{ MeV}^2/\text{c}^2$$

$$p_{n\text{-rms}} = \sqrt{< p_n^2 >} = 4.56 \times 10^{-24} \text{ kg m/s}$$
$$= 8.54 \times 10^{-3} \text{ MeV/c}$$

This value is again much too small to account for the measured transverse neutron momentum.

(3) Could the neutron momentum in the target result from the confinement of the neutrons inside an atomic nucleus and the Heisenberg Uncertainty Principle for spin-1/2 fermion neutrons?

The Uncertainty Principle tells us $(\Delta x)(\Delta p_x) > \frac{\hbar}{2} = \frac{h}{4\pi} = 3.29 \times 10^{-22}$ MeV s.

If we take Δx to be the diameter of the C-12 nucleus, then

$$\Delta x = 2(1.25 \times (12)^{1/3} \times 10^{-15} \text{ m}) = 5.7 \times 10^{-15} \text{ m},$$

and $\Delta p_x > (3.29 \times 10^{-22} \text{ MeV s})/(5.7 \times 10^{-15} \text{ m}) = 5.77 \times 10^{-8} \text{ MeV/(m/s)}$
or $\Delta p_x > 18$ MeV/c.

This value is the correct order of magnitude to account for the lower bound of the measured neutron motion.

(4) Could the neutron momentum in the target result from the Pauli Exclusion Principle applied to neutrons confined to the nucleus?

As a group of spin-1/2 fermion particles confined within the nucleus, the neutrons are subject not only to the Uncertainty Principle but also to the Pauli Exclusion Principle. As a result of their fermion character, no more than two neutrons (one spin-up, one spin-down) can occupy an energy level within the nucleus. Thus, as more neutrons are added to nuclei, the maximum energy and average energy of the neutrons increase. The difference between the highest and lowest energy level of the neutrons in the nucleus is the Fermi energy (E_F) of the neutrons. For the neutrons in a nucleus modeled as a gas of non-relativistic, non-interacting particles in an isotropic potential well, the Fermi energy is related to the number density ($N_n/V_{nucleus}$) of neutrons by the equation

$$E_F = \frac{\hbar^2}{2m_n}\left(\frac{3\pi^2 N}{V}\right)^{2/3}.$$

The neutron number density in C-12 is

$$\frac{N_n}{V} = \frac{A/2}{\frac{4}{3}\pi R^3} = \frac{6}{\frac{4}{3}\pi\left(1.25\text{x}10^{-15}\text{x}12^{1/3}\ \text{m}\right)^3} = 6.11 \text{ x } 10^{43}\ \text{m}^{-3}.$$

Then the Fermi energy for C-12 is

$$E_{FC\text{-}12} \cong 31\ \text{MeV}.$$

The corresponding Fermi momentum of the neutrons is

$$p_F = \sqrt{2m_n E_F} = \sqrt{2\left(939.6\ \frac{\text{MeV}}{c^2}\right)(31\ \text{MeV})} = 241\ \text{MeV}/c.$$

This value, in fact, corresponds to a typical value of the transverse neutron momentum measured in the MINERvA data.

The Fermi velocity for the neutrons at the Fermi surface is

$$v_F = p_F/m_n = 0.26\ c = 7.7 \text{ x } 10^7\ \text{m/s}.$$

The Fermi temperature at which thermal effects become comparable to quantum effects for the neutrons in C-12 is

$$T_F = E_F/k_B = 3.6 \text{ x } 10^{11}\ \text{K}.$$

Clearly, quantum effects are dominant here!