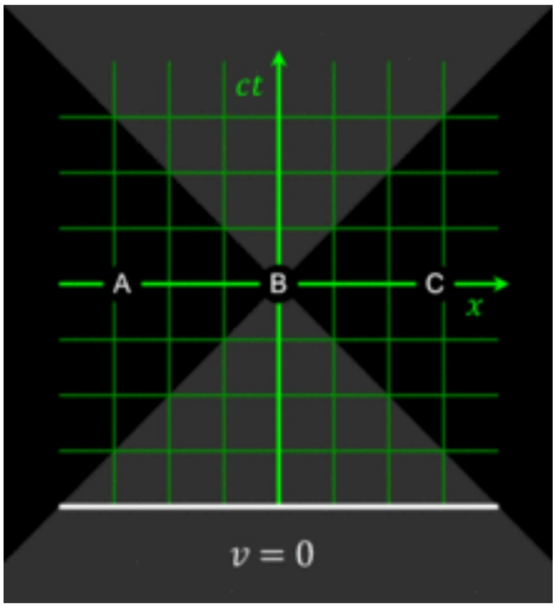
Spacetime Diagrams

-What is a spacetime diagram?

-How do you draw a spacetime diagram for an object moving with a constant velocity?

-What's the benefit of representing relativity this way?



Light moves at c.

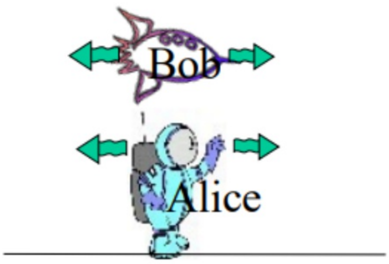
Maxwell's equations

Michelson-Morley experiment

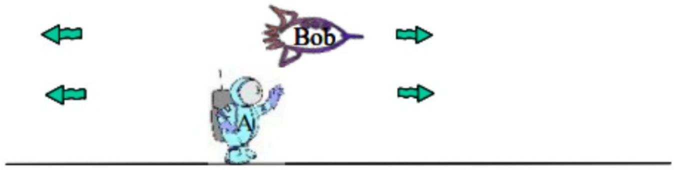
regardless of your reference frame

<u>Reason:</u> Alice is on the ground. Bob is in a high speed rocket (2c/3) relative to Alice. When Bob is directly above Alice, both of them send out laser pulses in each direction.

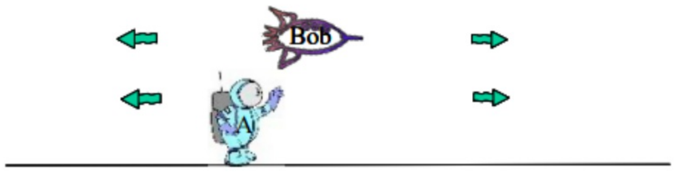
Which cartoon corresponds to the laser pulses a moment later?



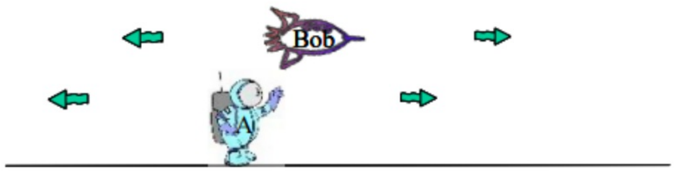
1)



2)



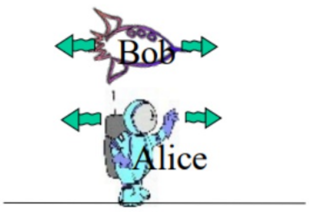
3)



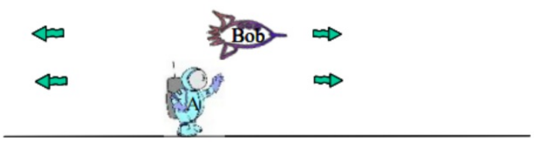
4) None of these.

Reason: Alice is on the ground. Bob is in a high speed rocket (2c/3) relative to Alice. When Bob is directly above Alice, both of them send out laser pulses in each direction.

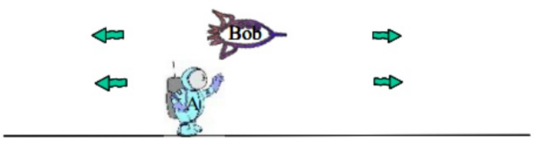
Which cartoon corresponds to the laser pulses a moment later?



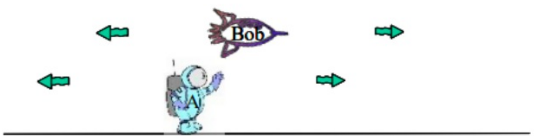
1)



2)



3)



4) None of these.

Answer: It depends!

Reason: Alice is on the ground. Bob is at 0.67c relative to Alice. Each have two clocks they want to trigger with light pulses. All the clocks *are on towers*, equidistant from Alice. When Bob is directly above Alice, they both send their signals .

Which sets of clocks will be synchronized?

1) A & B

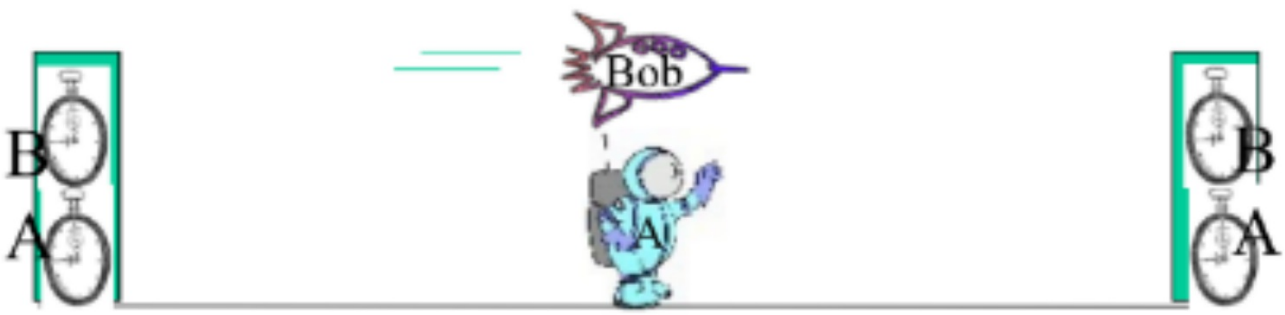
2) Only A

3) Only B

4) Neither

5) It depends on who you ask.

6) Can't be determined.



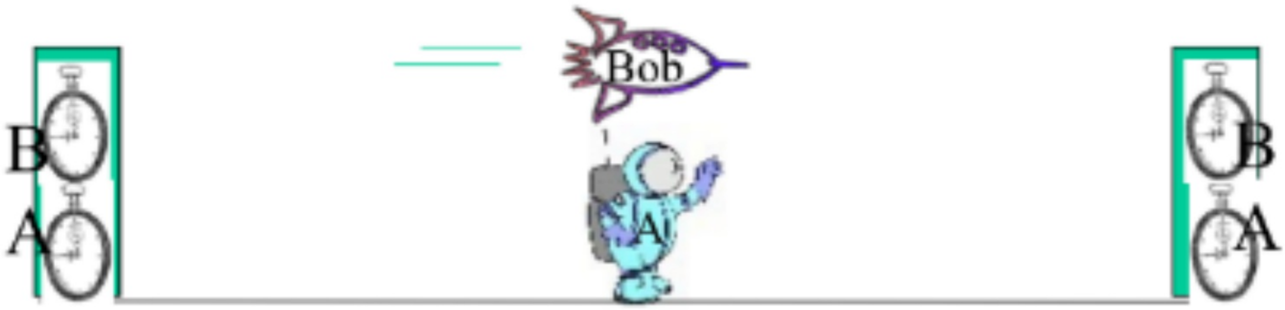
Reason: Alice is on the ground. Bob is at 0.67c relative to Alice. Each have two clocks they want to trigger with light pulses. All the clocks *are on towers*, equidistant from Alice. When Bob is directly above Alice, they both send their signals .

Which sets of clocks will be synchronized?

5) It depends on who you ask.

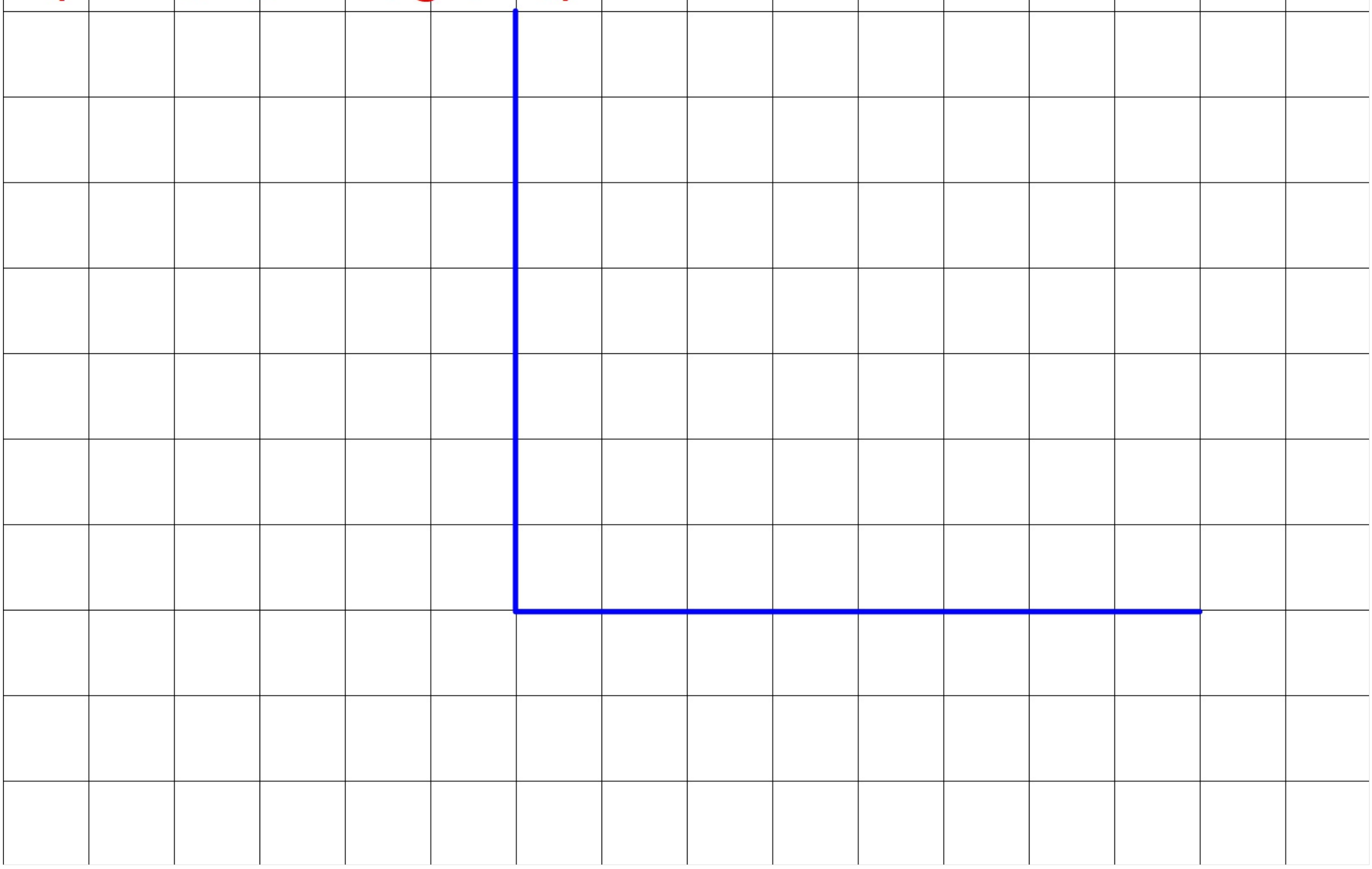
Alice says: Both A & B

Bob says: Neither

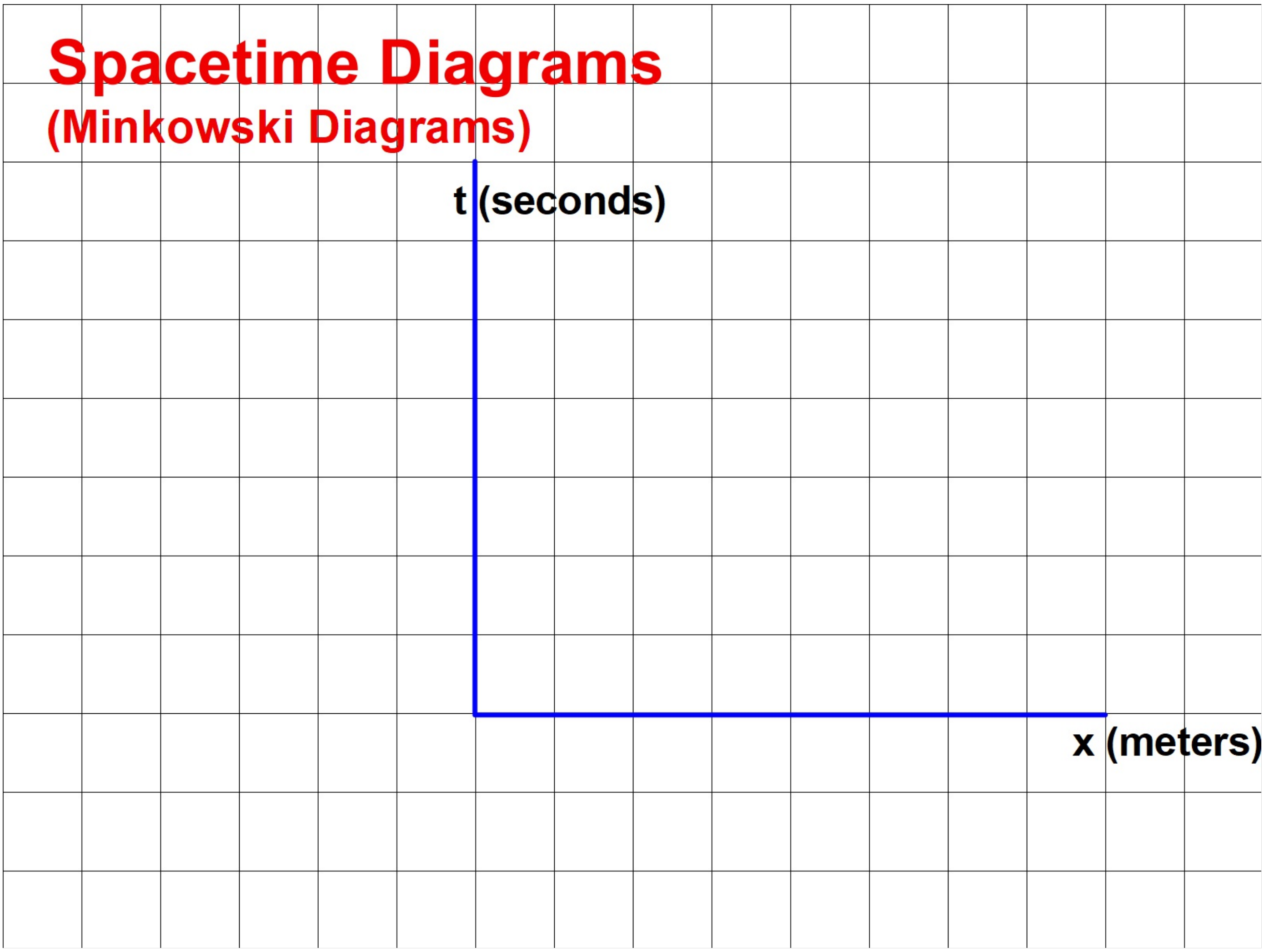


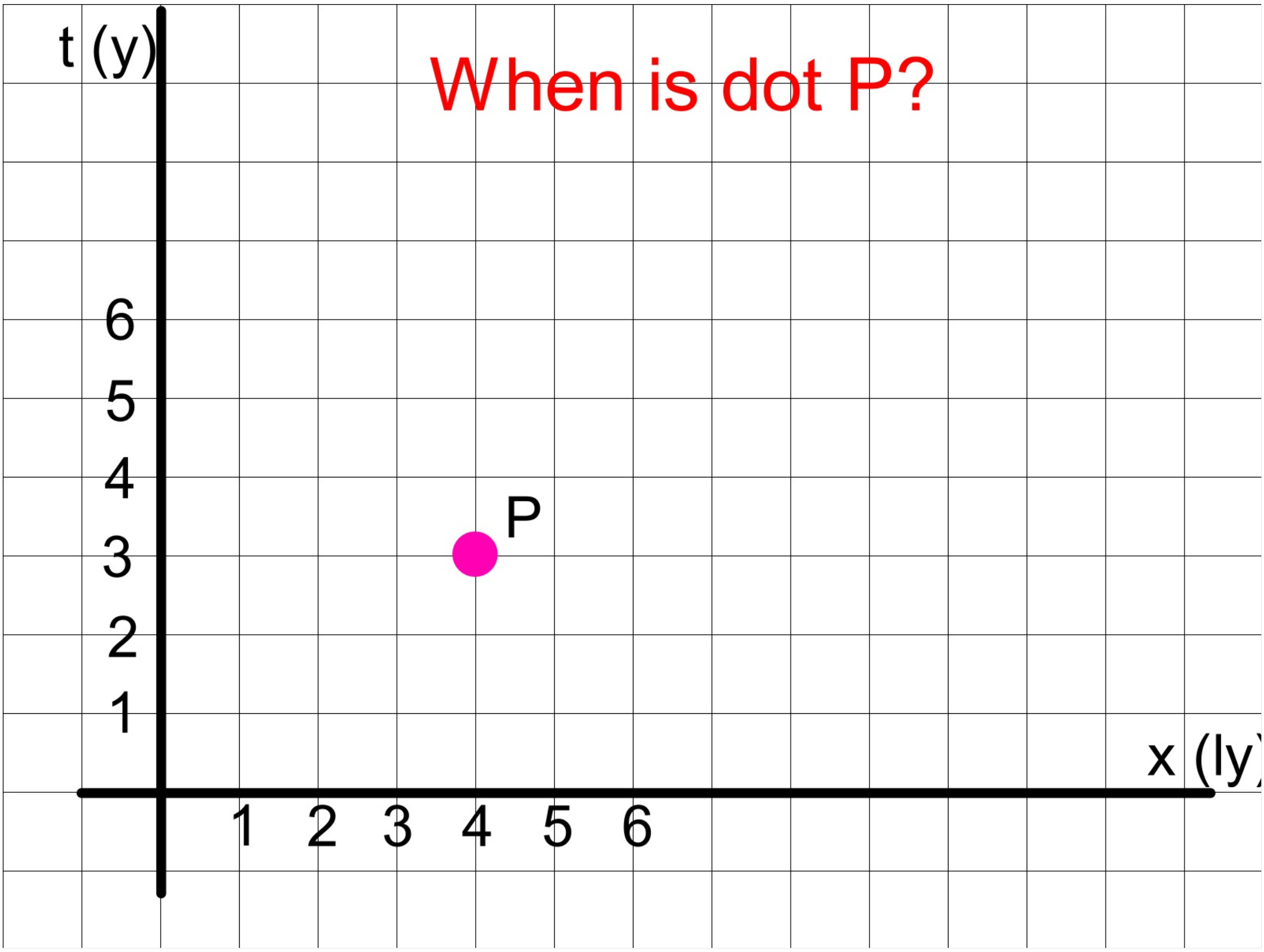
Spacetime Diagrams

(Minkowski Diagrams)

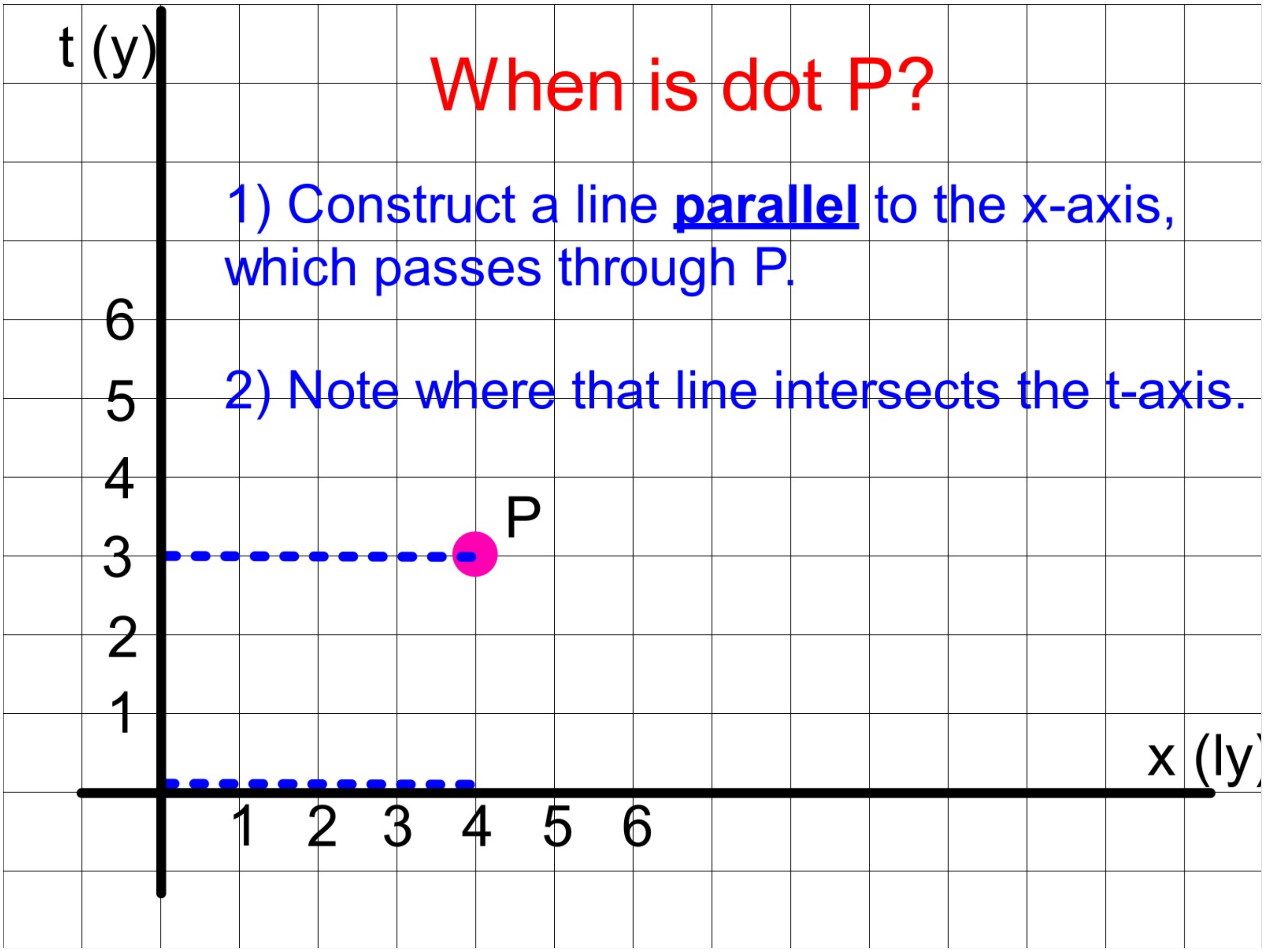


Spacetime Diagrams
(Minkowski Diagrams)

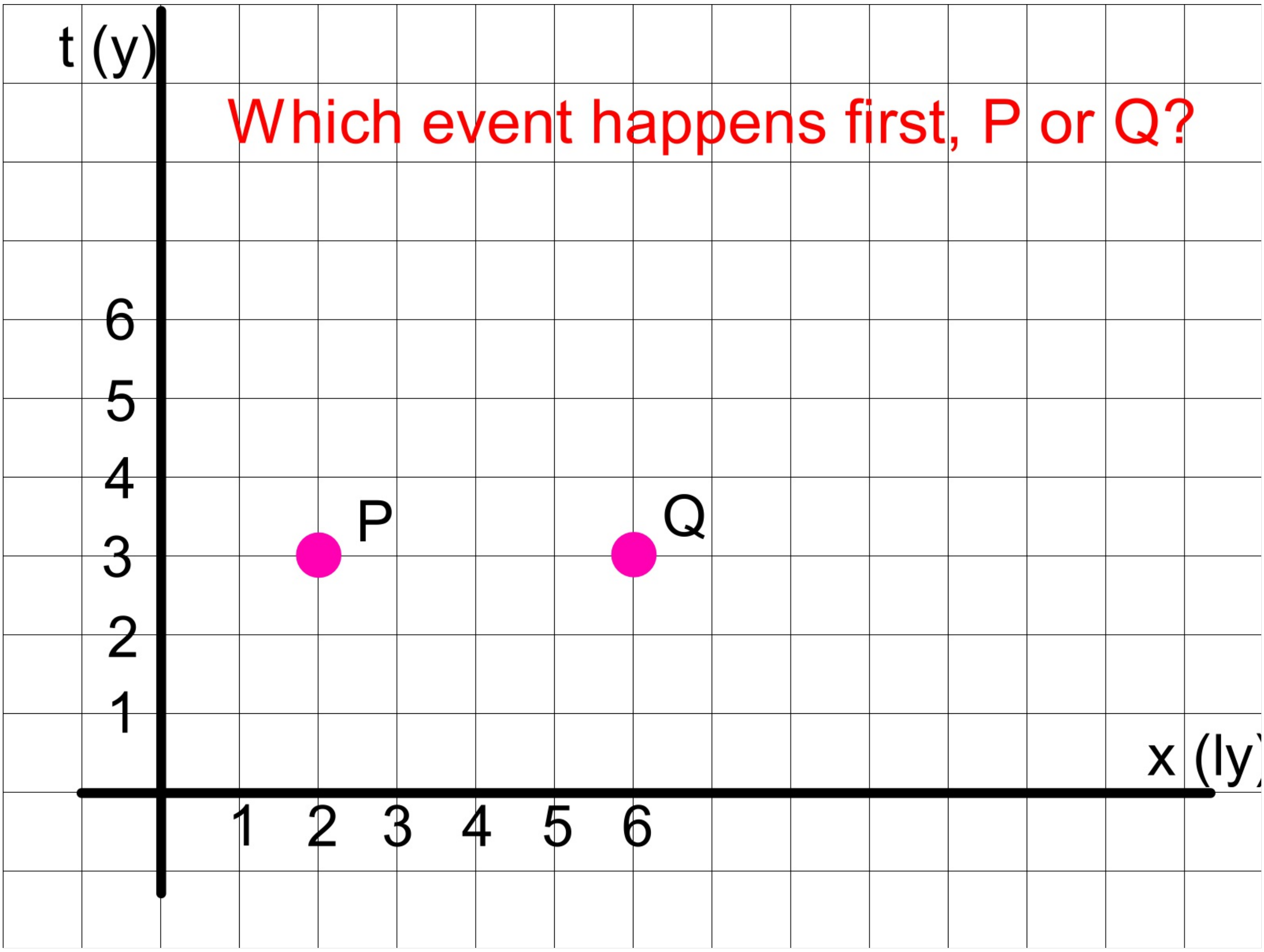


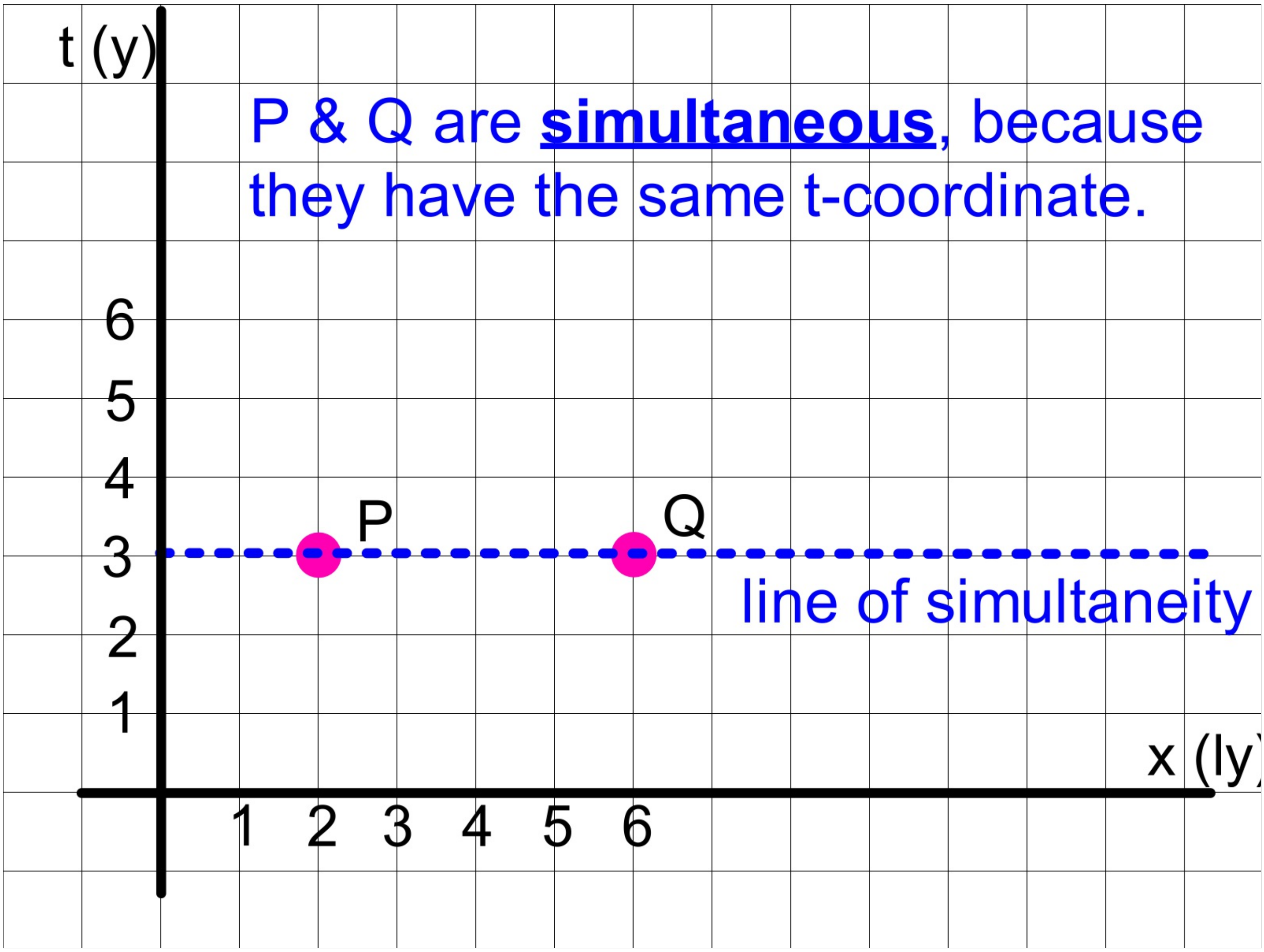


When is dot P?
t (y)
x (ly)
P
1
2
3
4
5
6
1
2
3
4
5
6

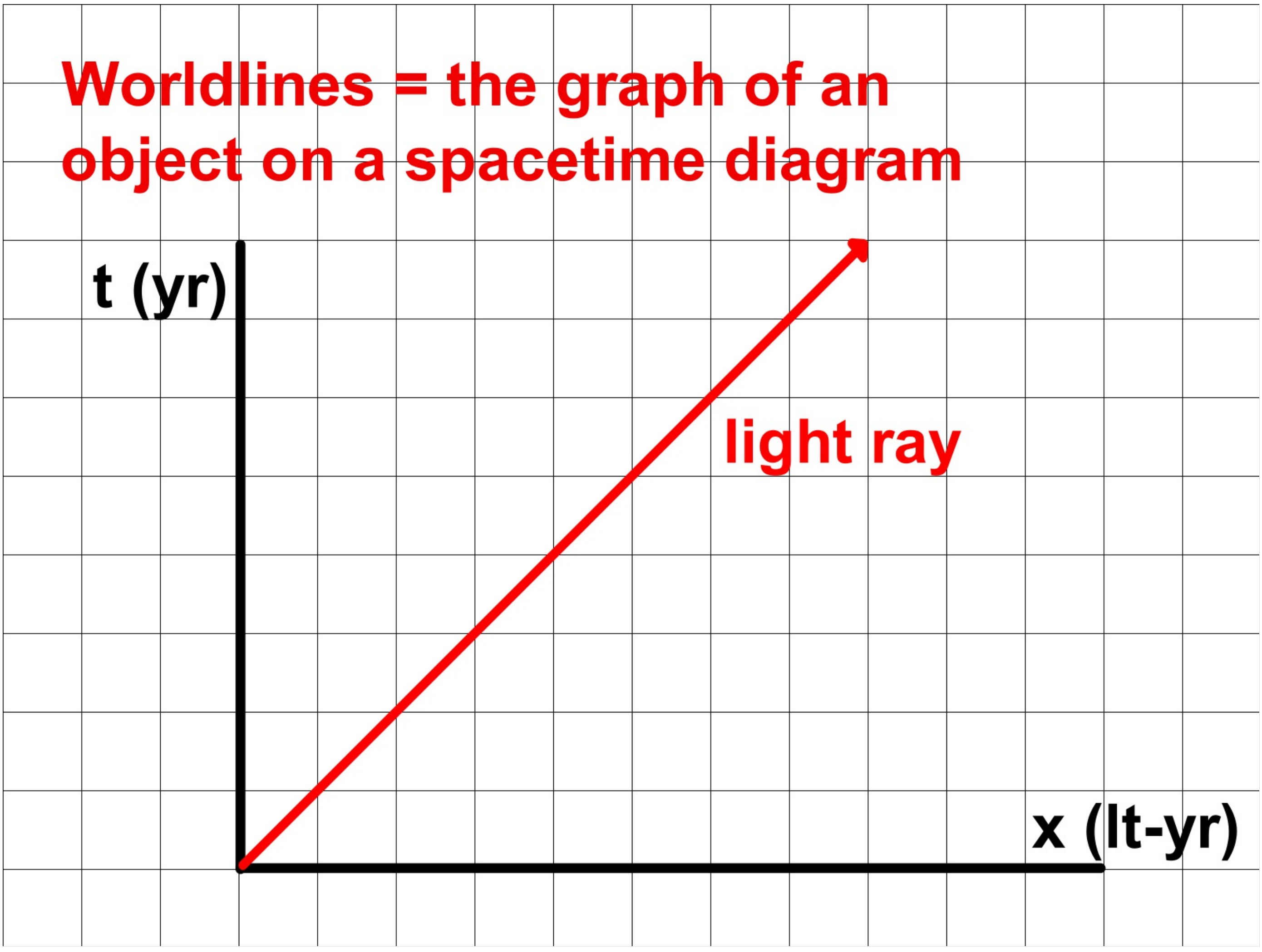


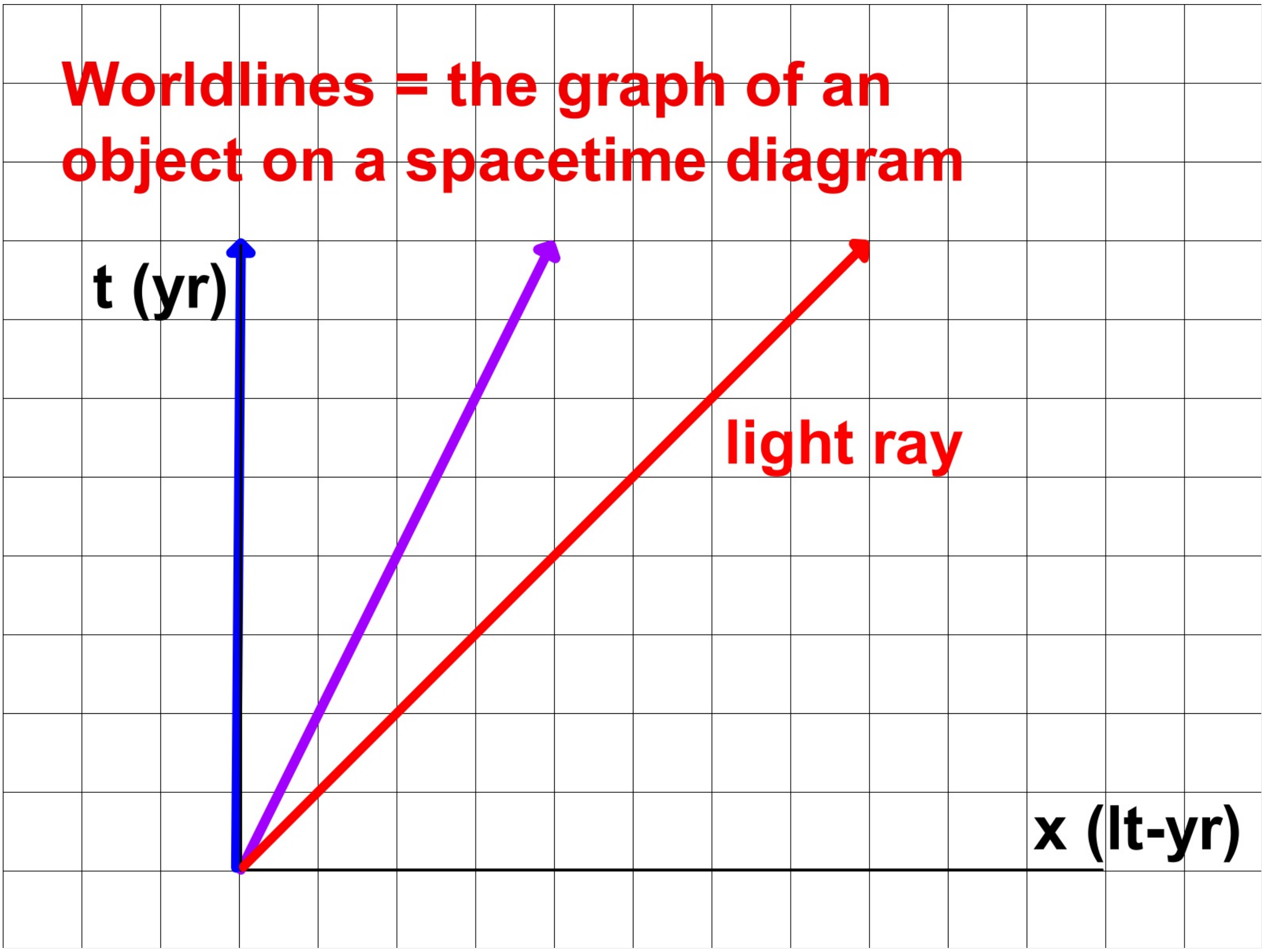
t (y)
When is dot P?
1) Construct a line parallel to the x-axis, which passes through P.
2) Note where that line intersects the t-axis.
6
5
4
3
2
1
P
x (ly)
1 2 3 4 5 6

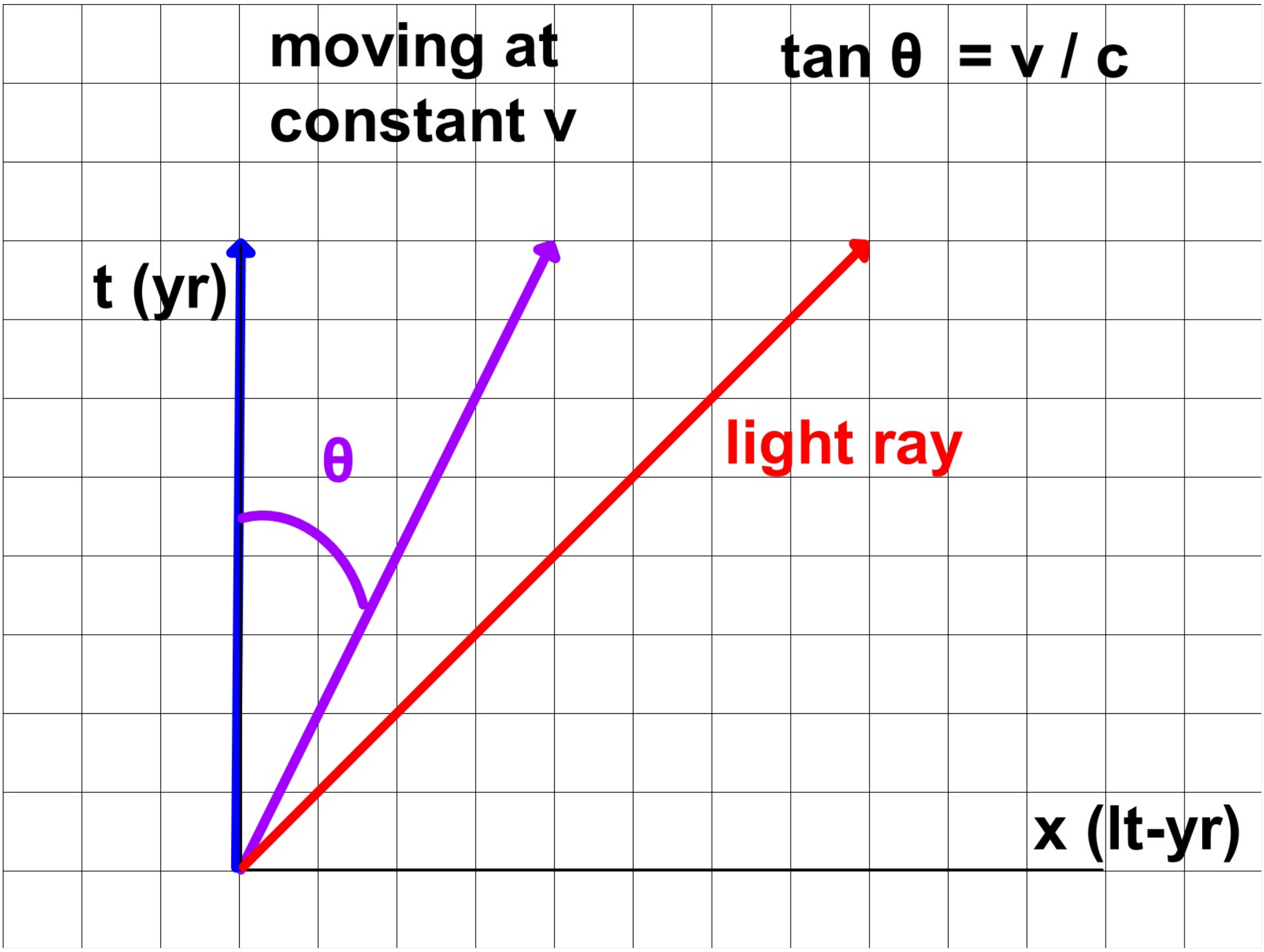

t (y)
Which event happens first, P or Q?
6
5
4
3
2
1
P
Q
x (ly)
1 2 3 4 5 6


t (y)
P & Q are **simultaneous**, because
they have the same t-coordinate.
6
5
4
3
2
1
P
Q
line of simultaneity
x (ly)
1 2 3 4 5 6

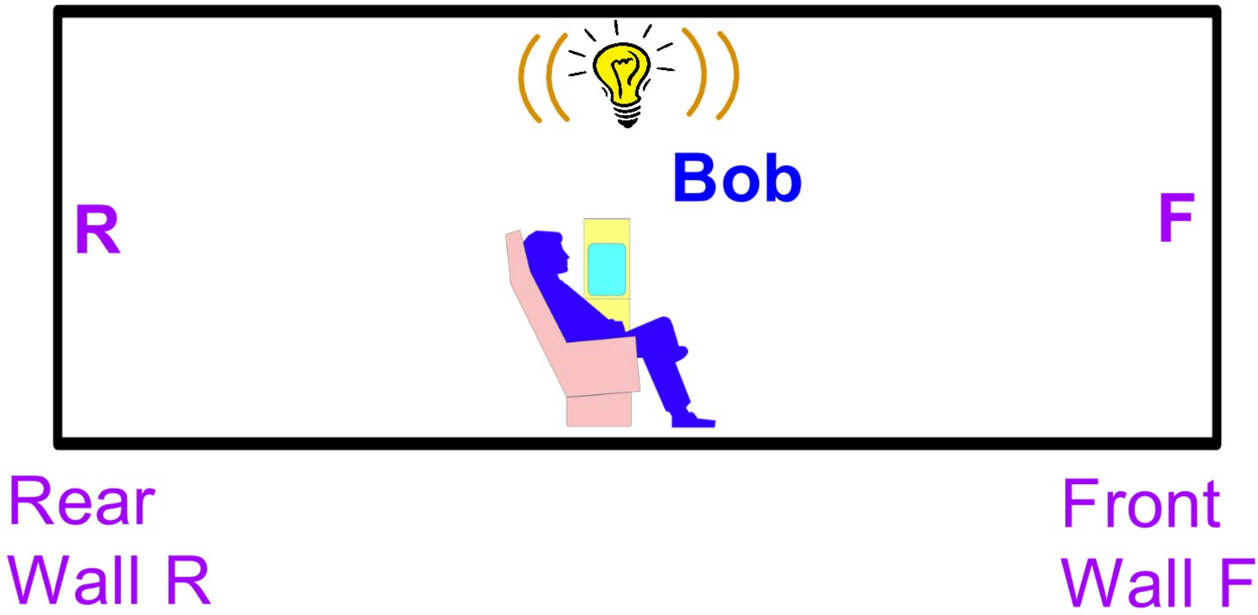
Worldlines = the graph of an object on a spacetime diagram

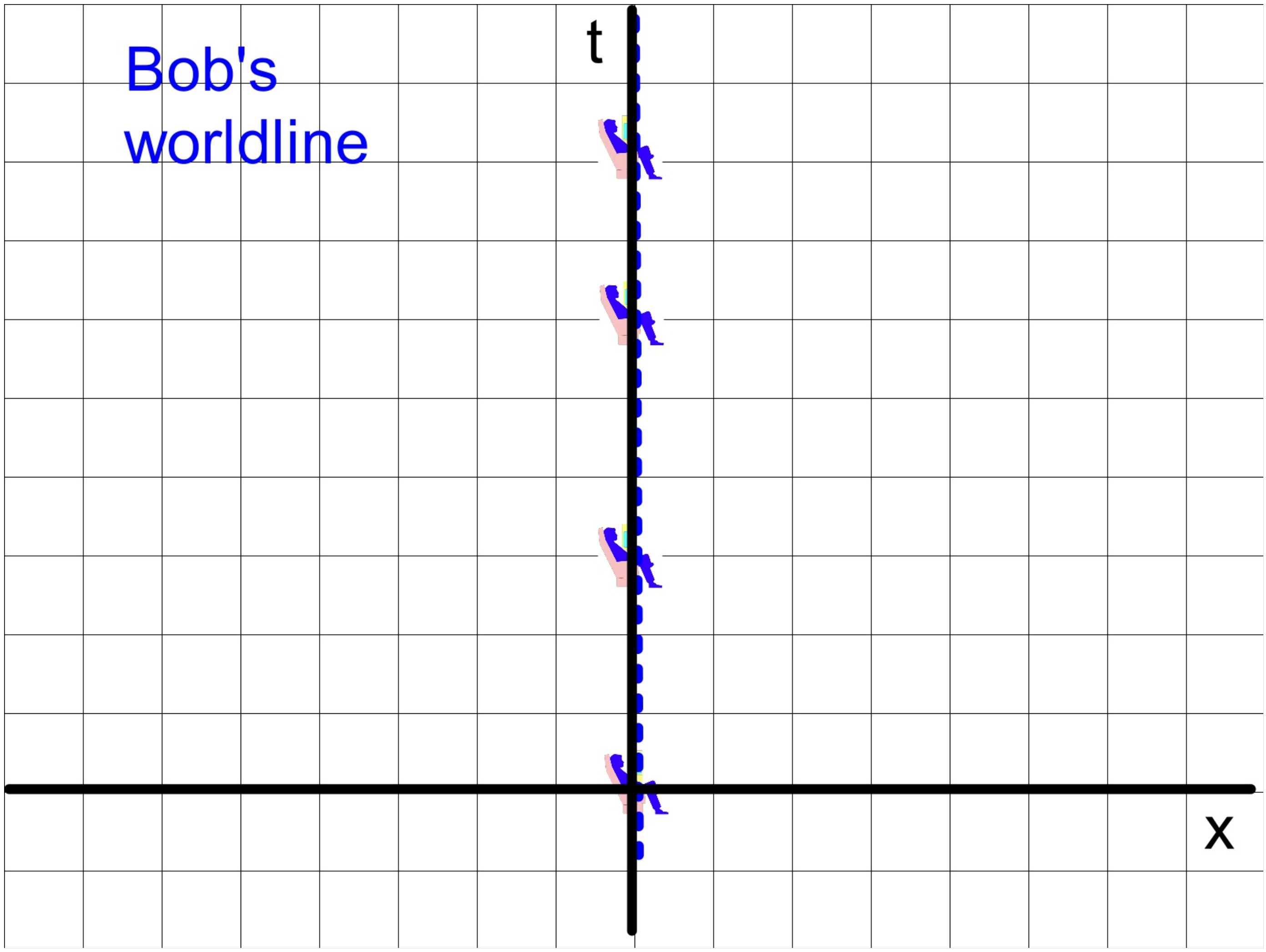

Worldlines = the graph of an
object on a spacetime diagram
t (yr)
light ray
x (lt-yr)

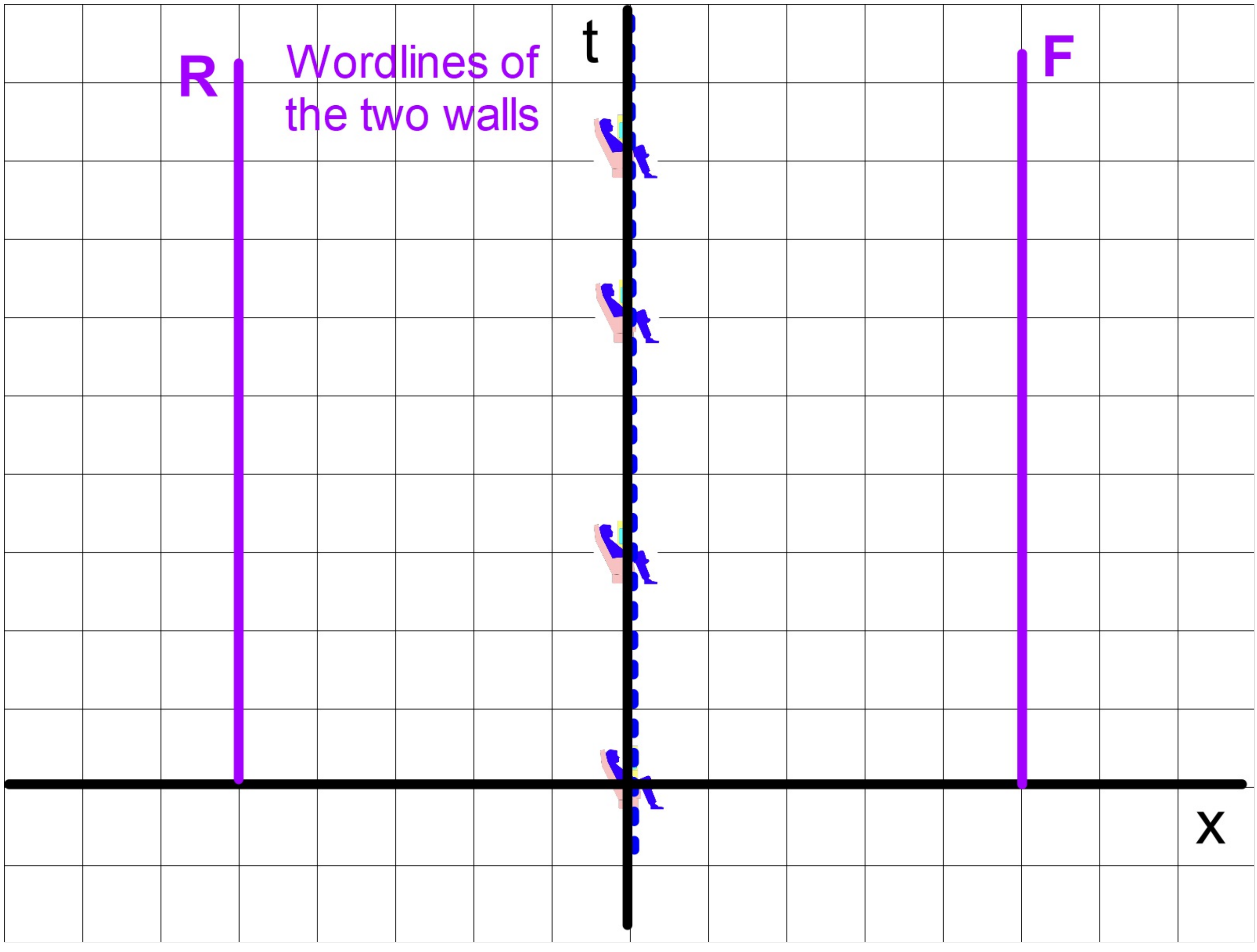

Worldlines = the graph of an object on a spacetime diagram
t (yr)
light ray
x (lt-yr)


moving at
constant v
tan θ = v / c
t (yr)
θ
light ray
x (lt-yr)

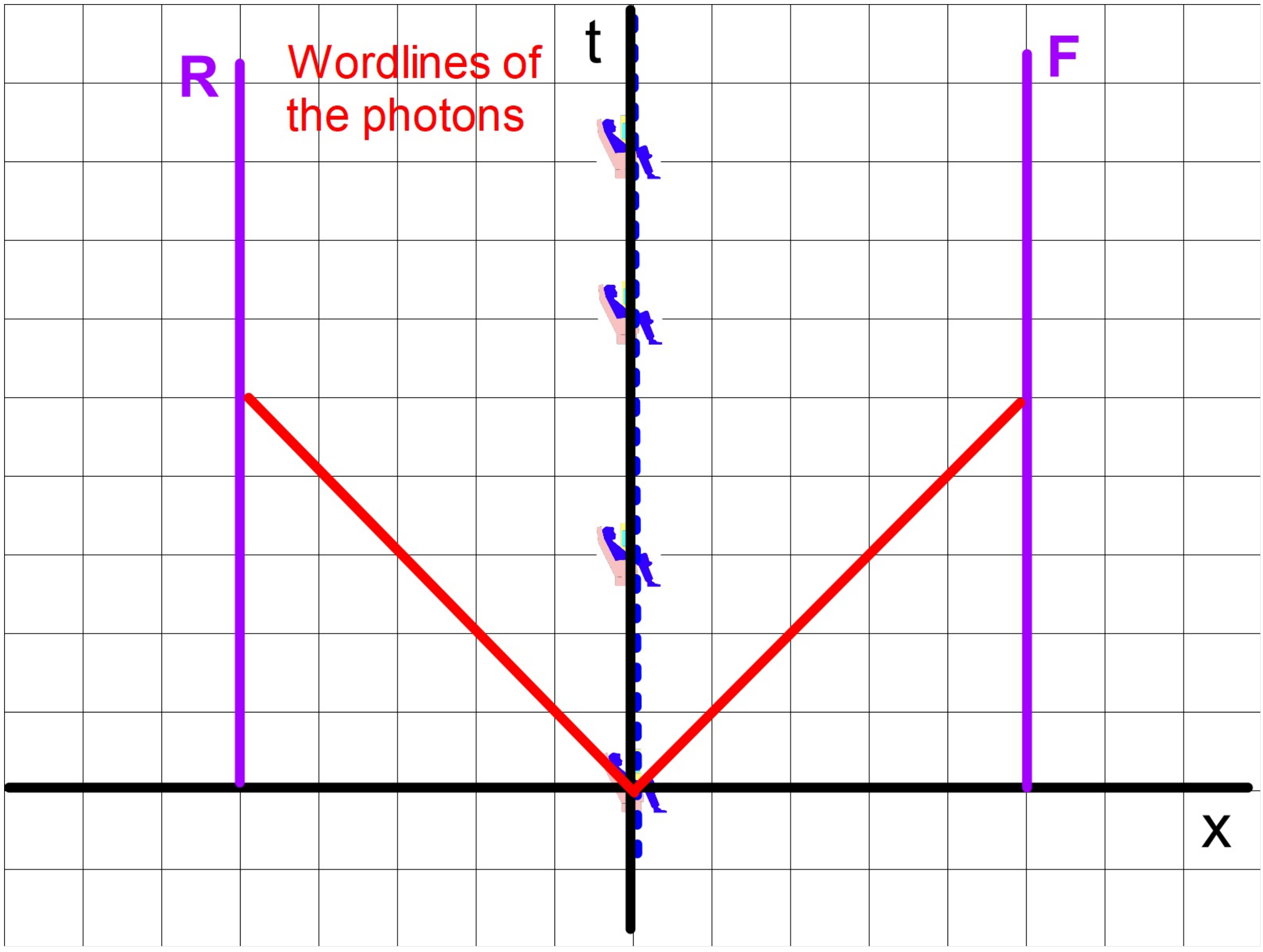
Bob at rest, relative to room.

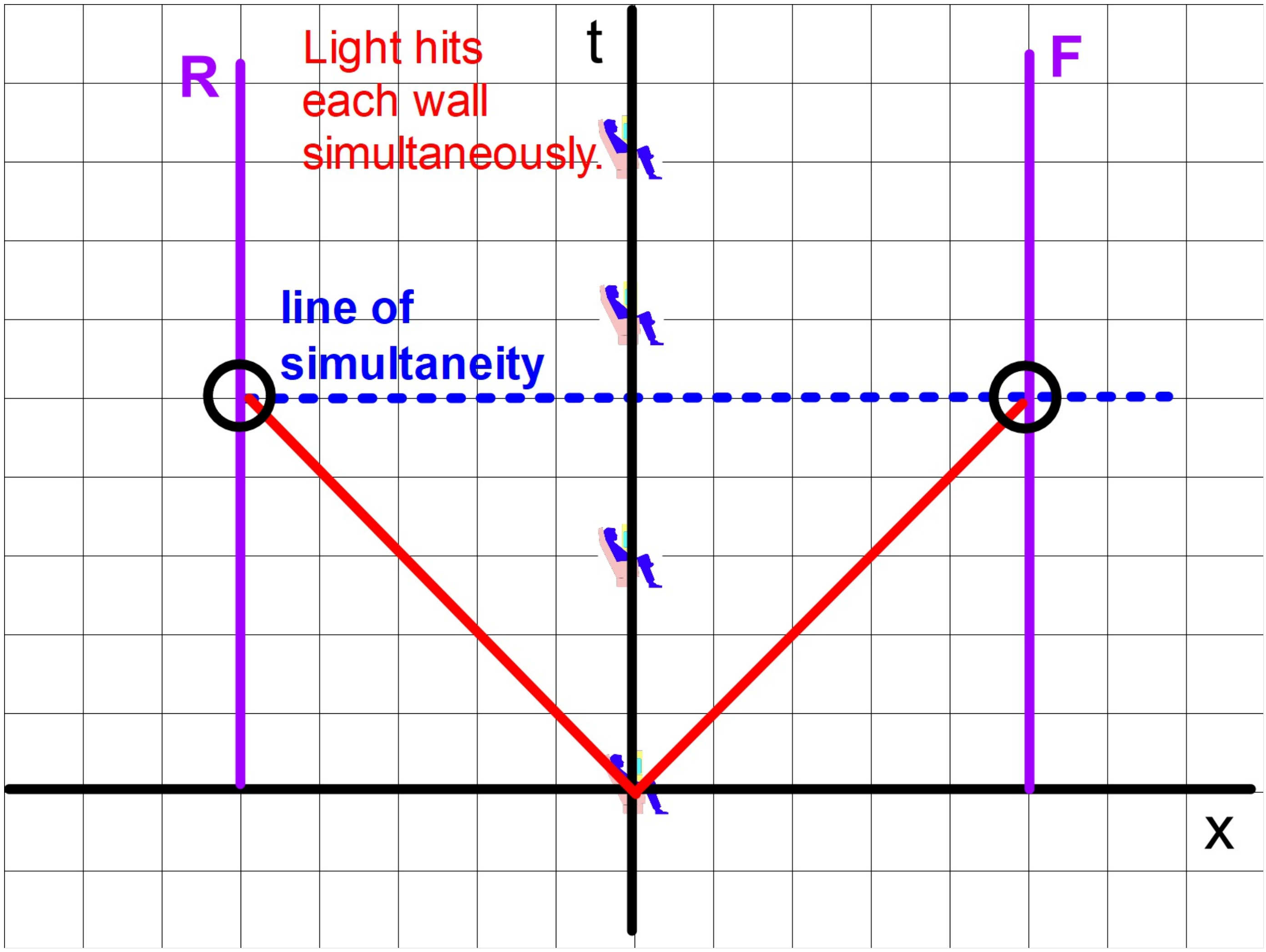


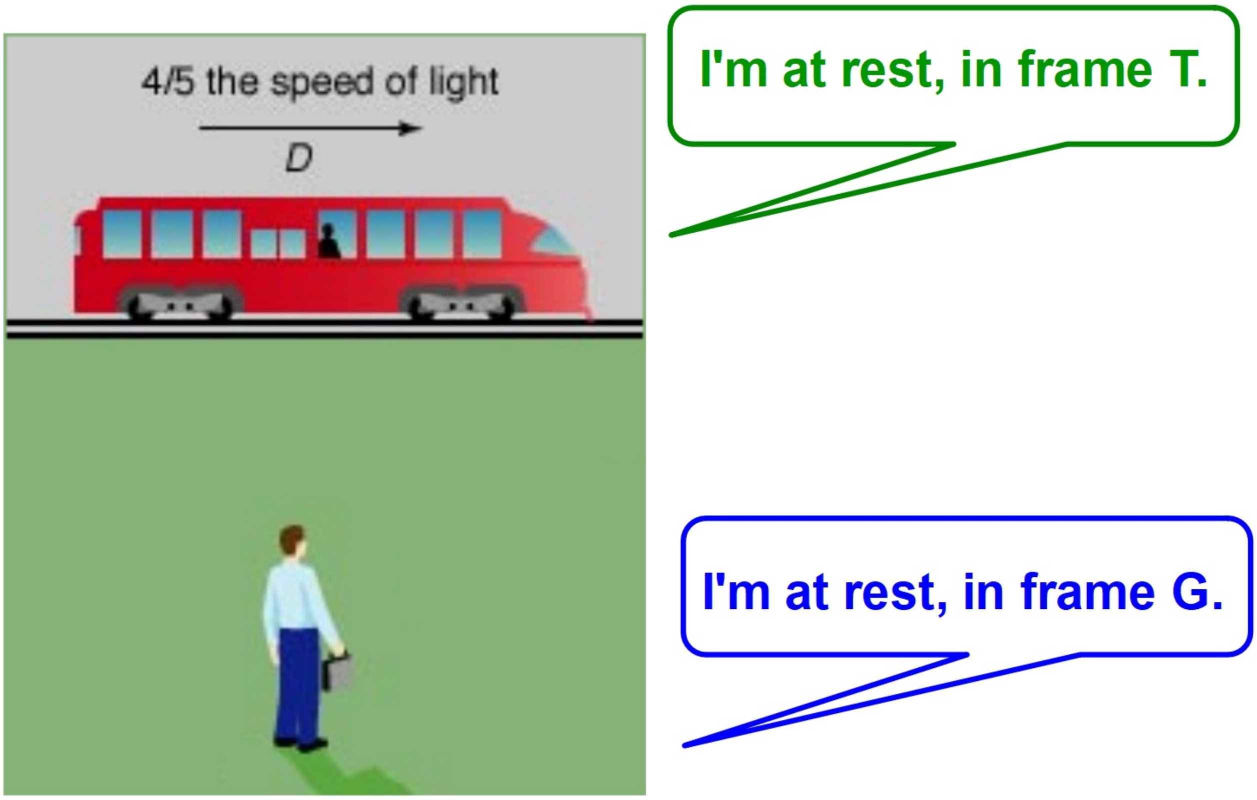

Bob's
worldline
t
x

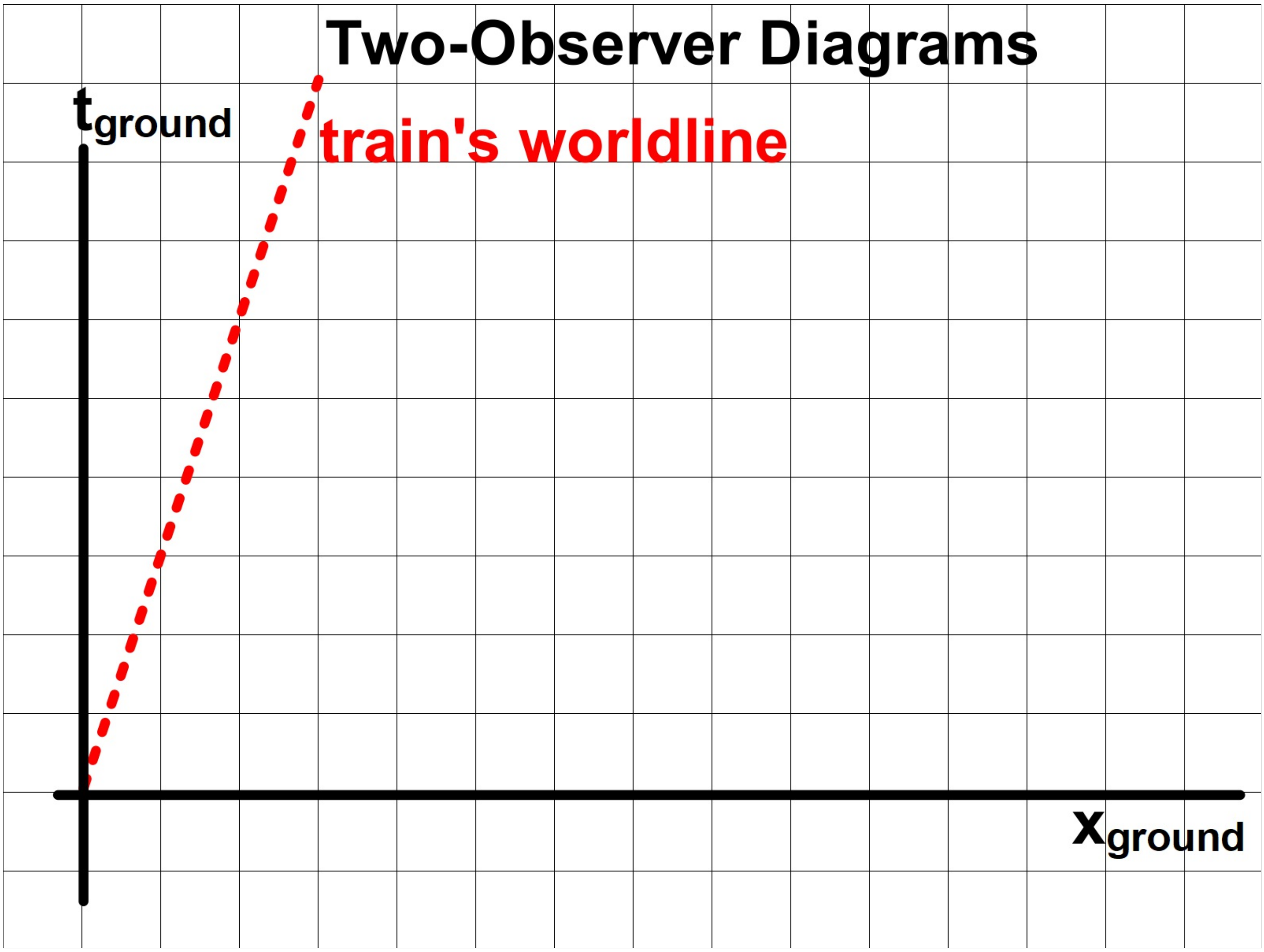


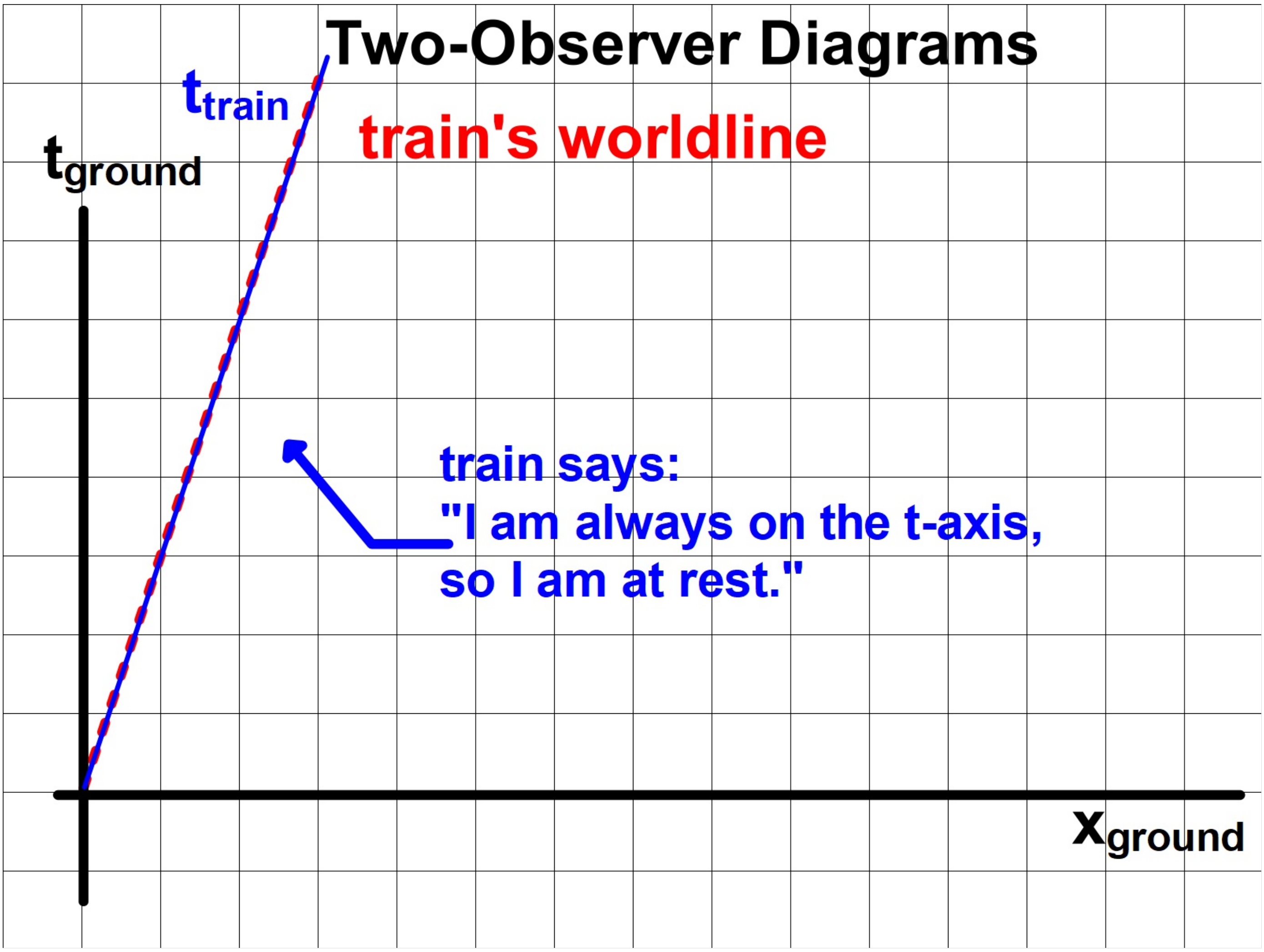
t
R
Wordlines of
the two walls
F
x


R
Wordlines of
the photons
t
F
x

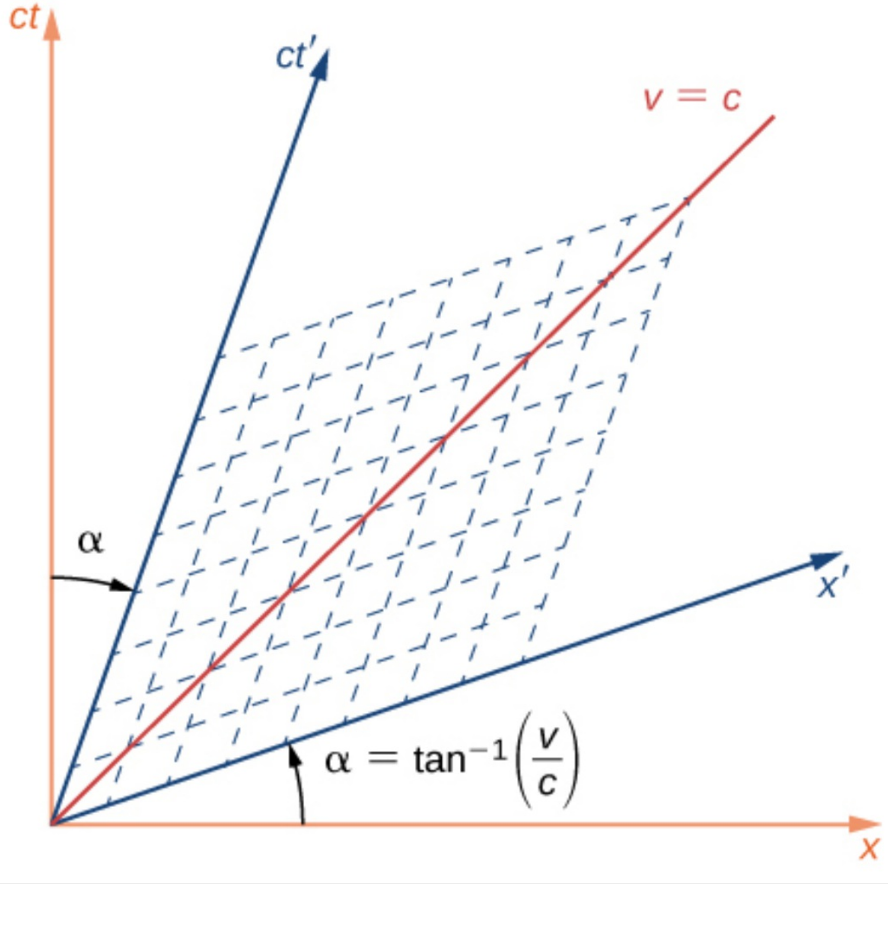

t
R
Light hits
each wall
simultaneously.
F
line of
simultaneity
x

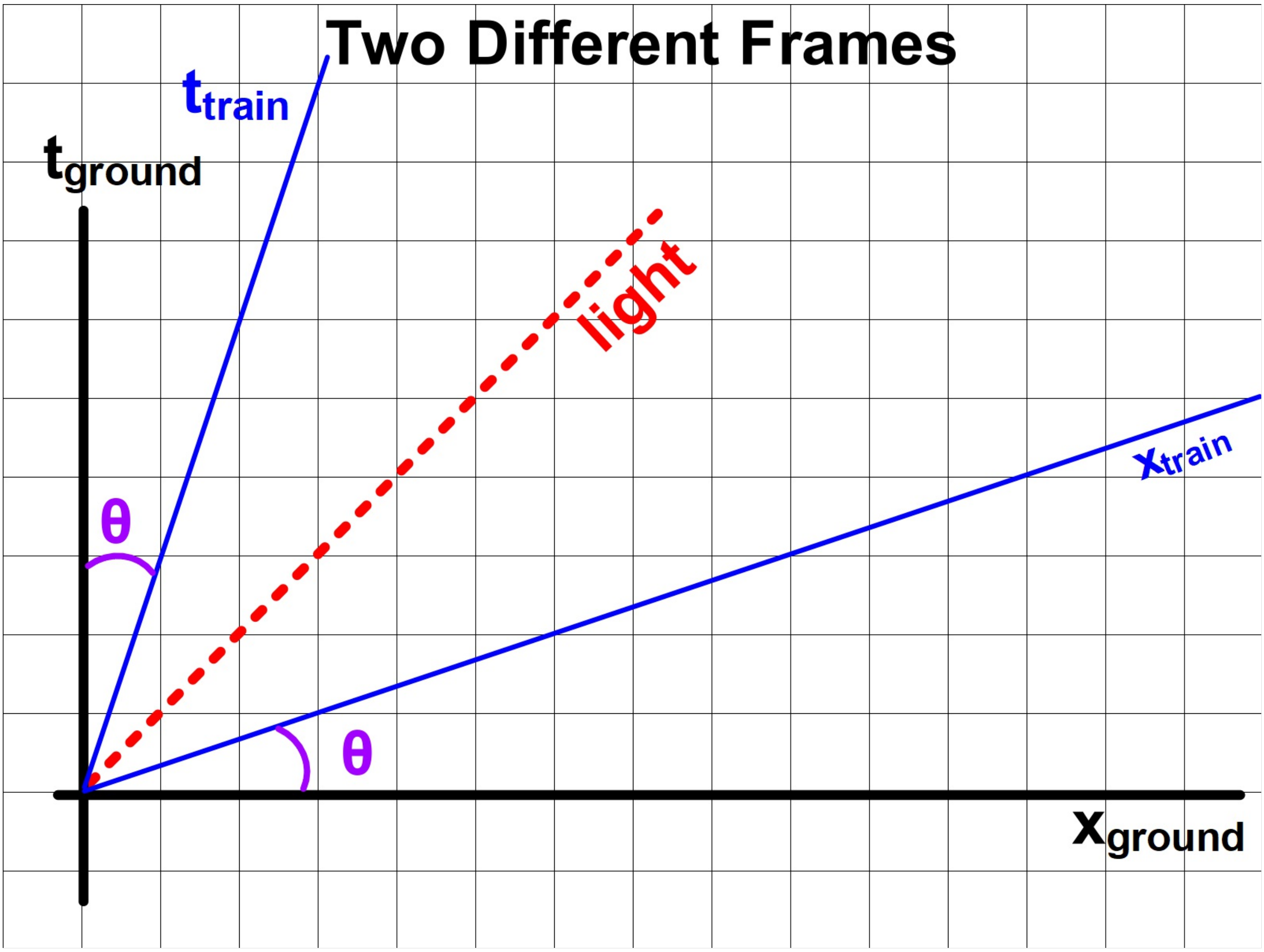

4/5 the speed of light
D
I'm at rest, in frame T.
I'm at rest, in frame G.

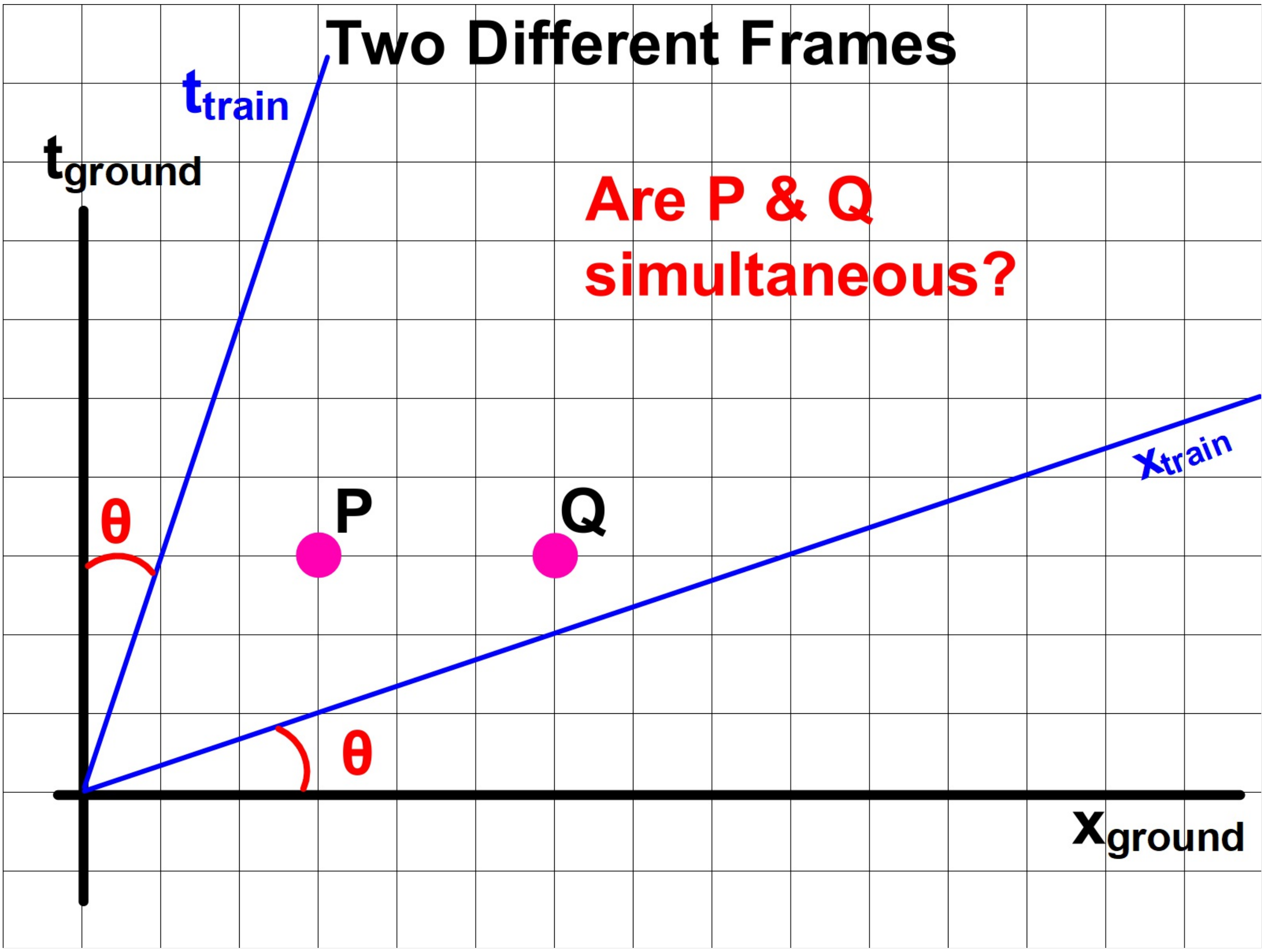

Two-Observer Diagrams
t_{ground}
train's worldline
x_{ground}

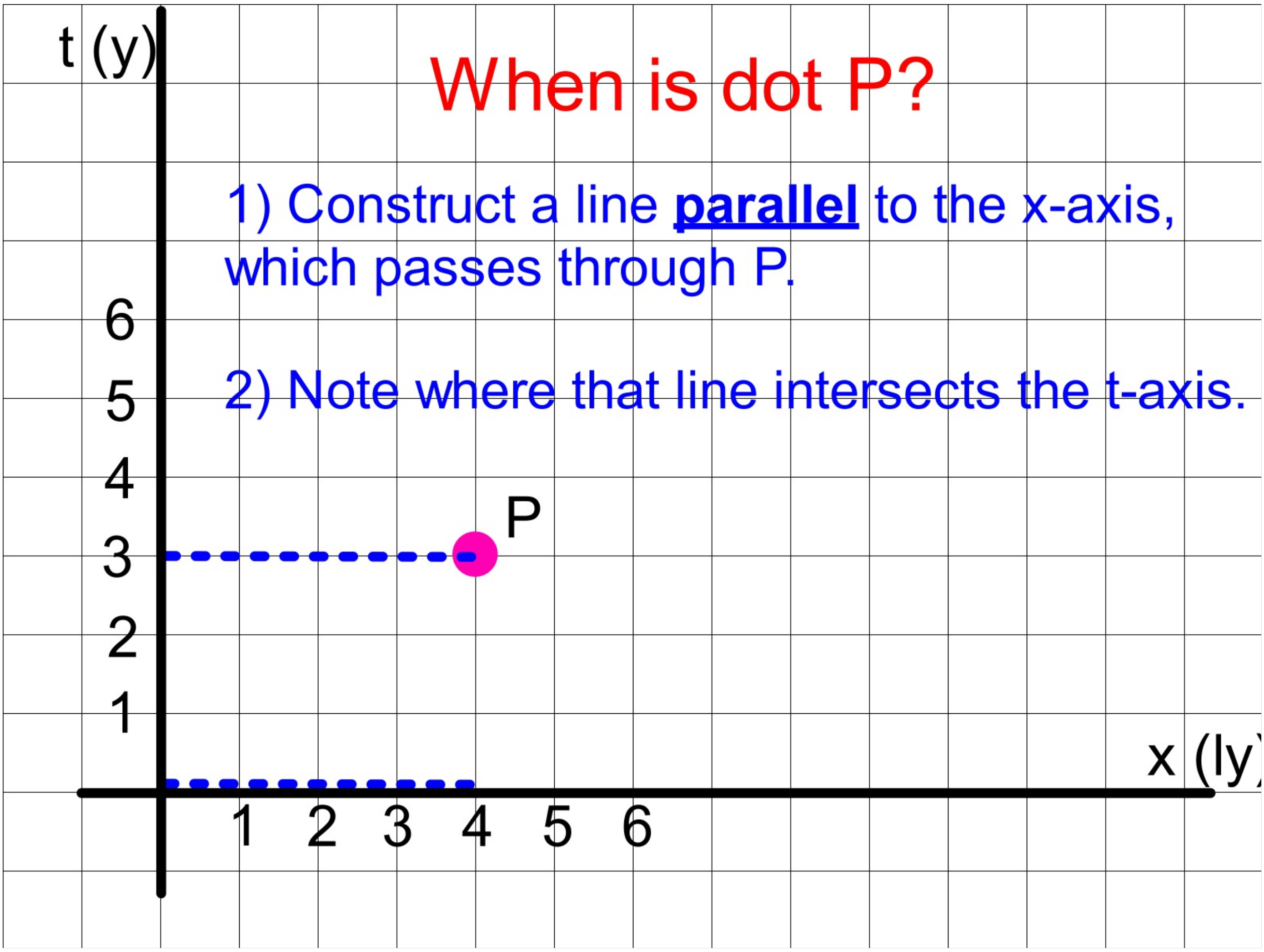

Two-Observer Diagrams
t_{train}
train's worldline
t_{ground}
train says:
"I am always on the t-axis,
so I am at rest."
x_{ground}

Two-Observer Diagrams

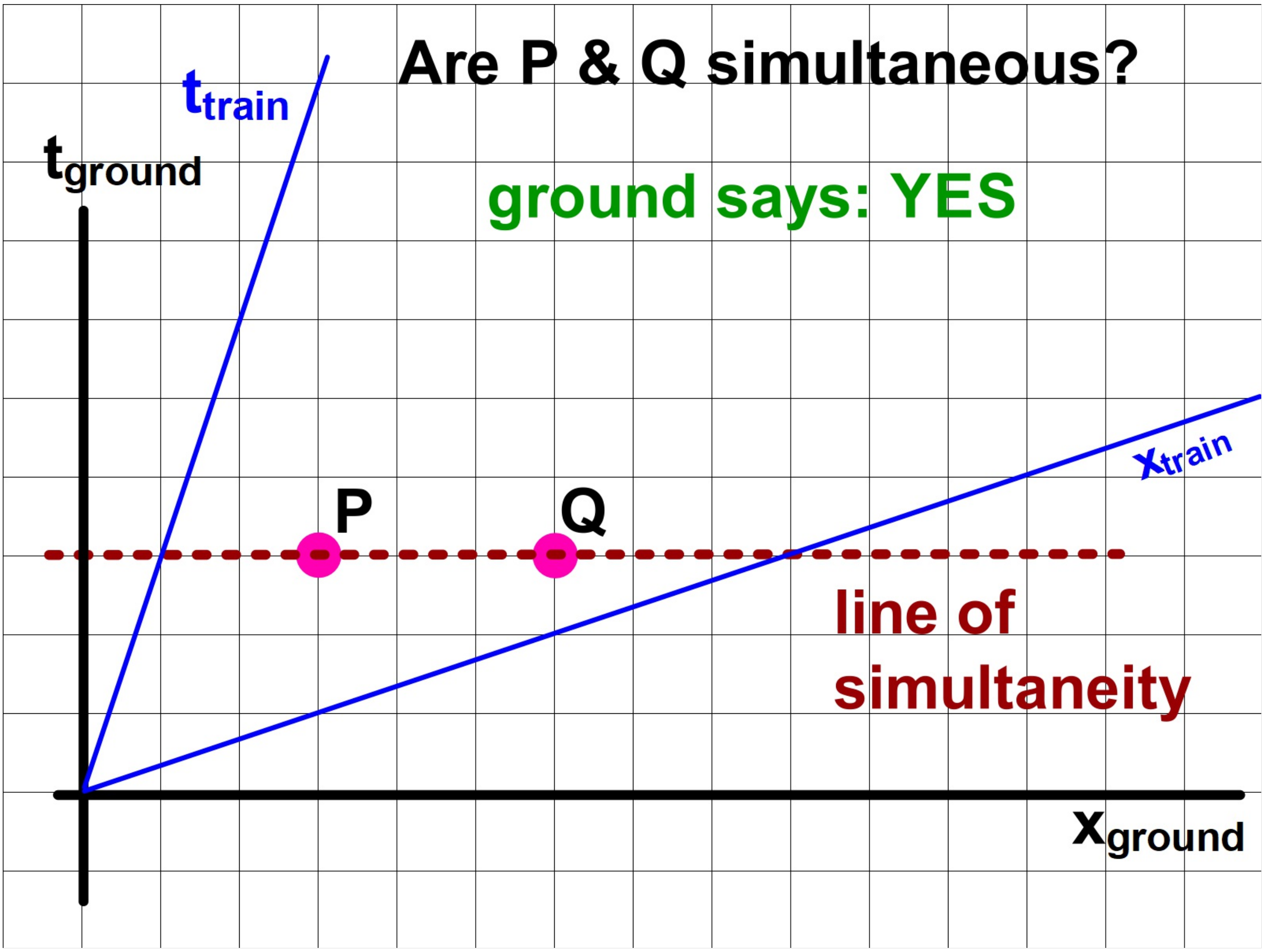


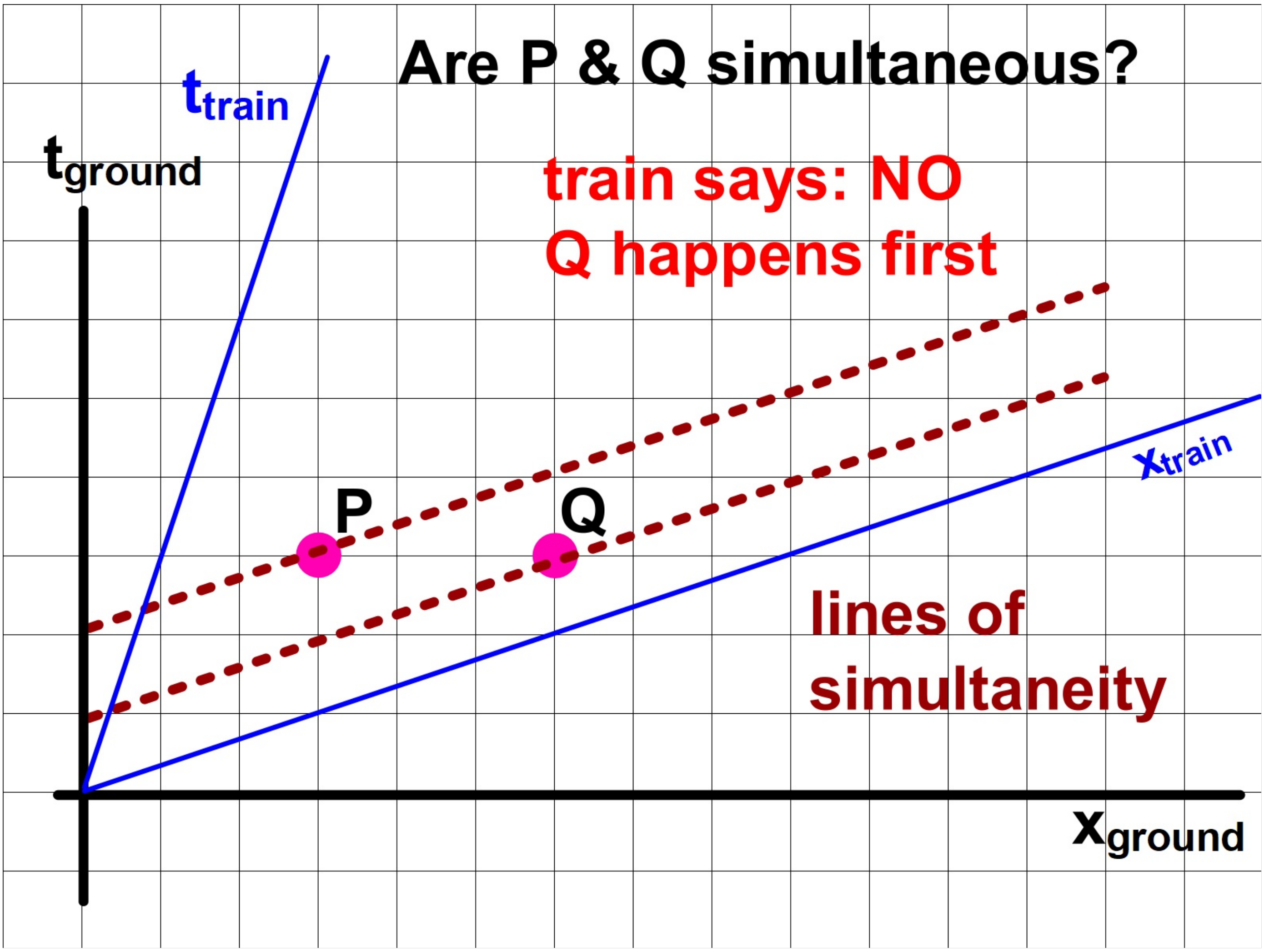

Two Different Frames
t_{train}
t_{ground}
light
θ
θ
x_{train}
x_{ground}


Two Different Frames
t_{train}
t_{ground}
Are P & Q simultaneous?
x_{train}
θ
P
Q
θ
x_{ground}

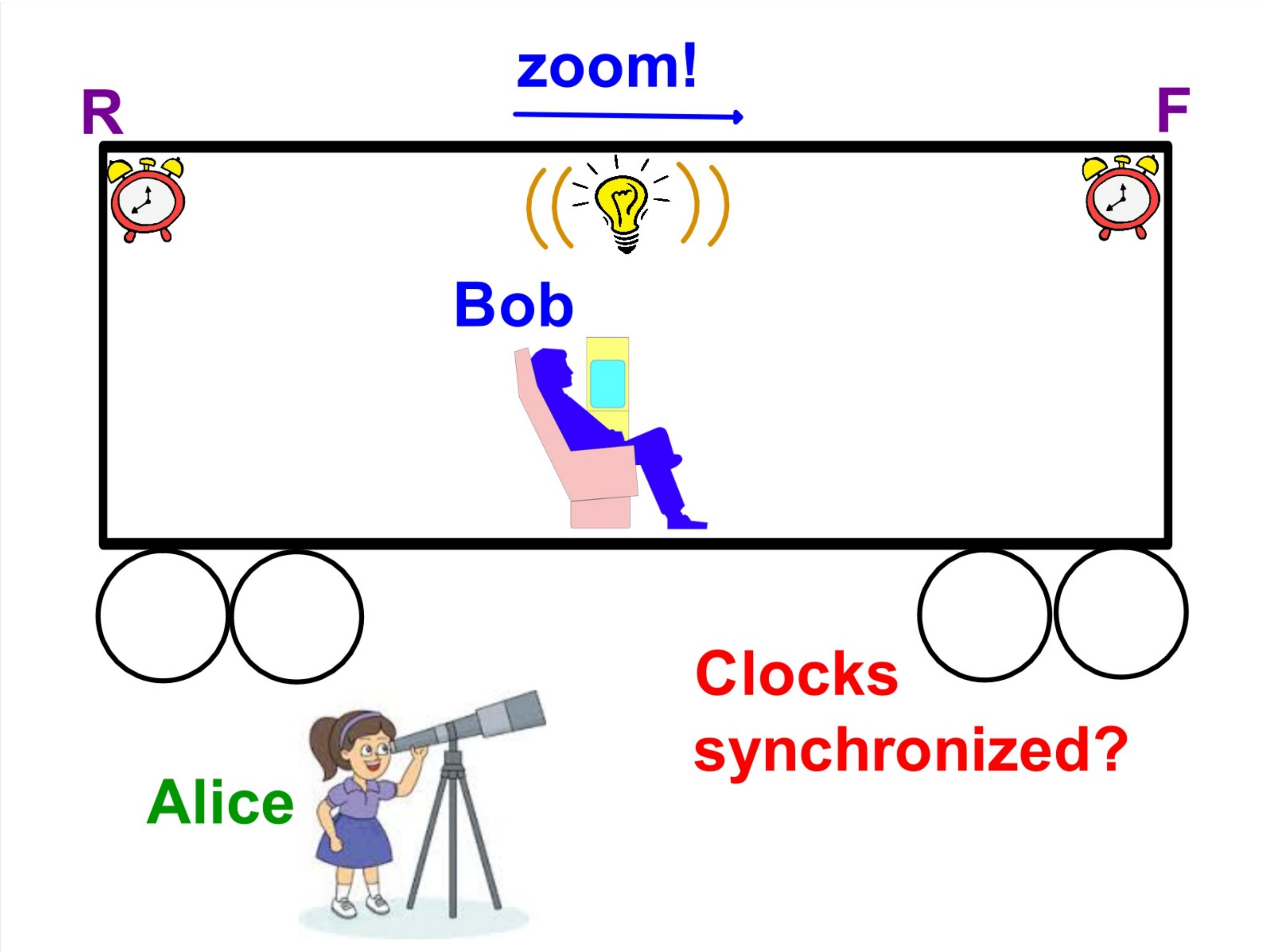


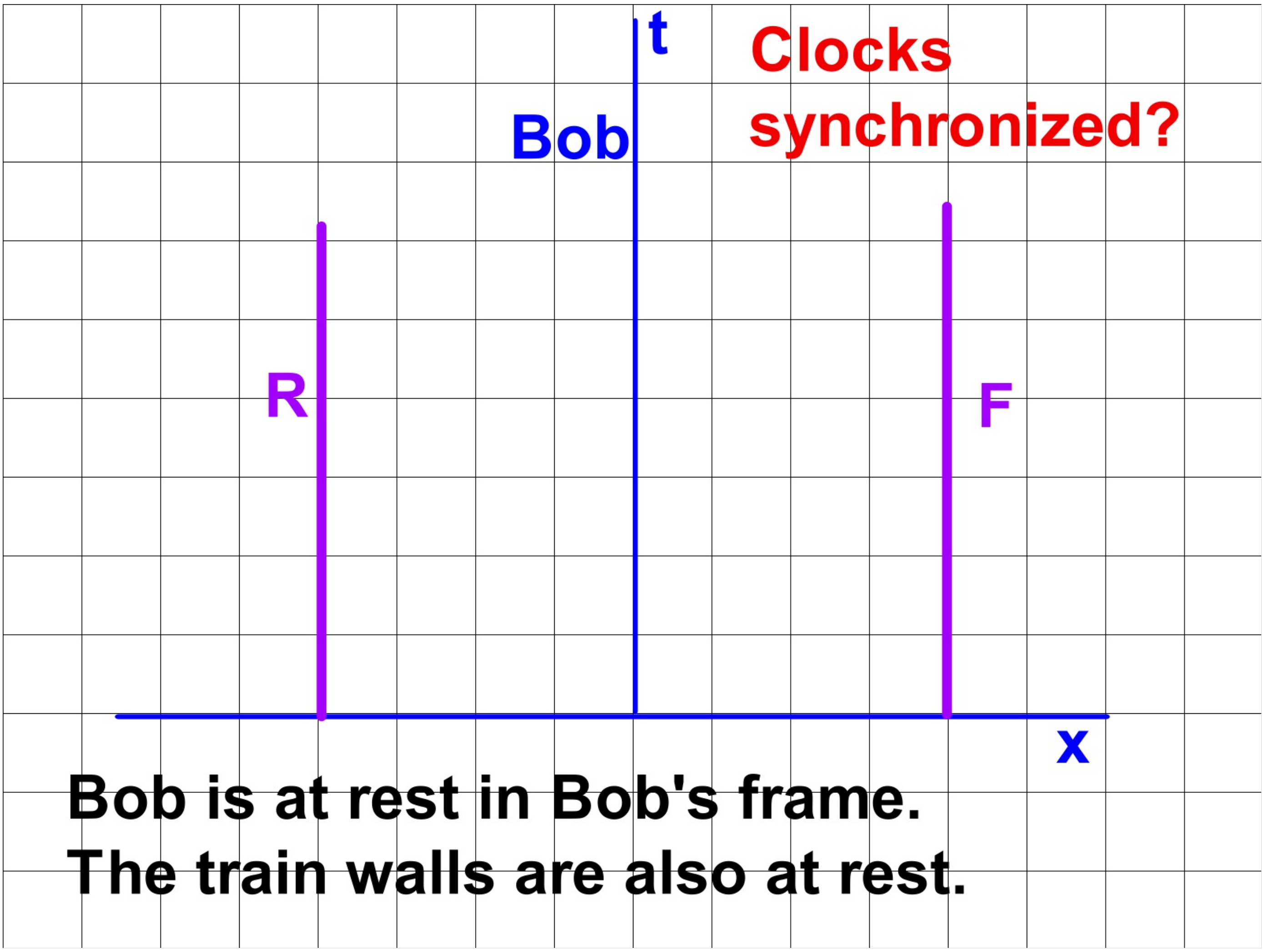
When is dot P?
1) Construct a line parallel to the x-axis, which passes through P.
2) Note where that line intersects the t-axis.
t (y)
x (ly)
P
1
2
3
4
5
6
1
2
3
4
5
6

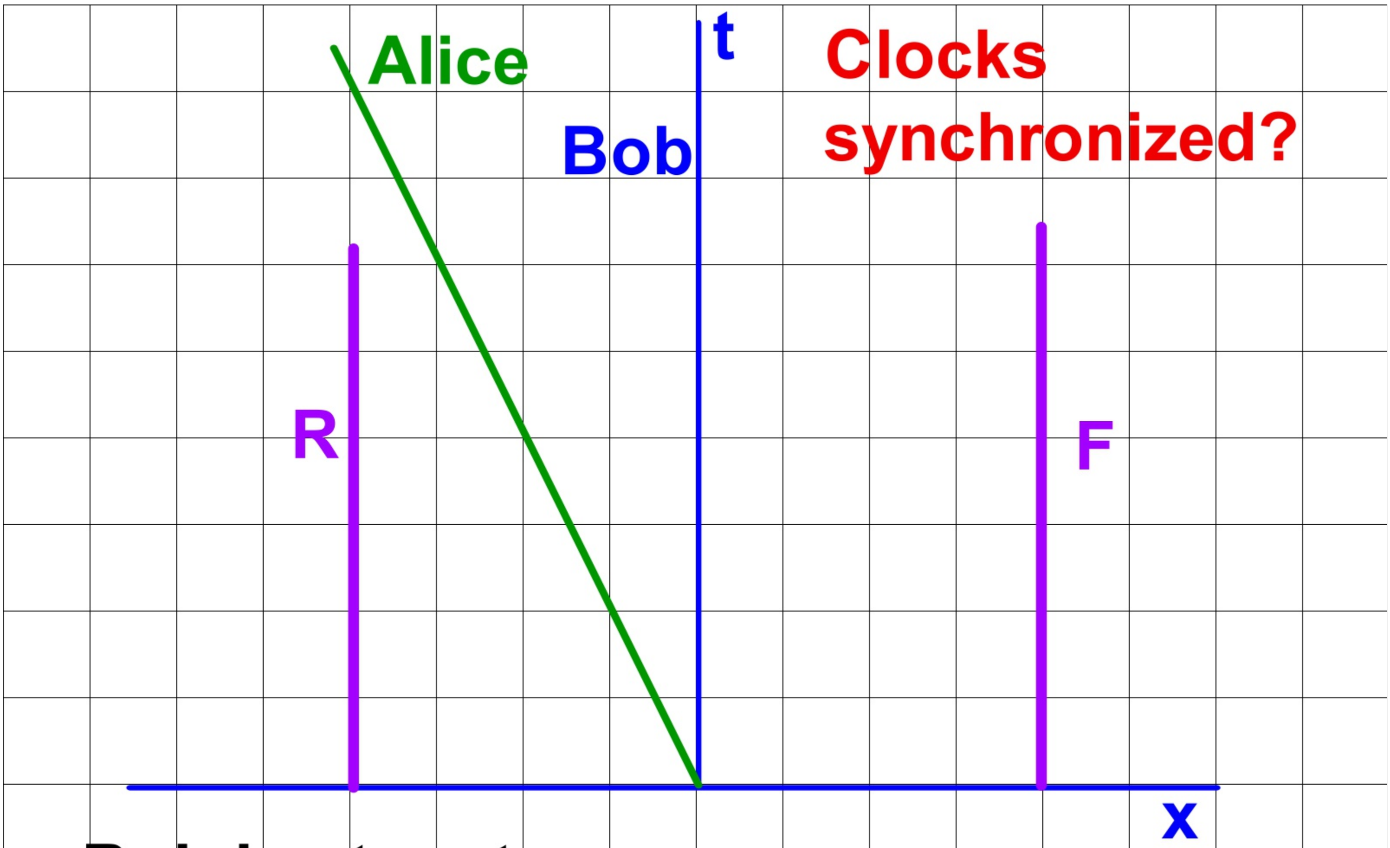

Are P & Q simultaneous?
ground says: YES
t_{train}
t_{ground}
x_{train}
P
Q
line of
simultaneity
x_{ground}



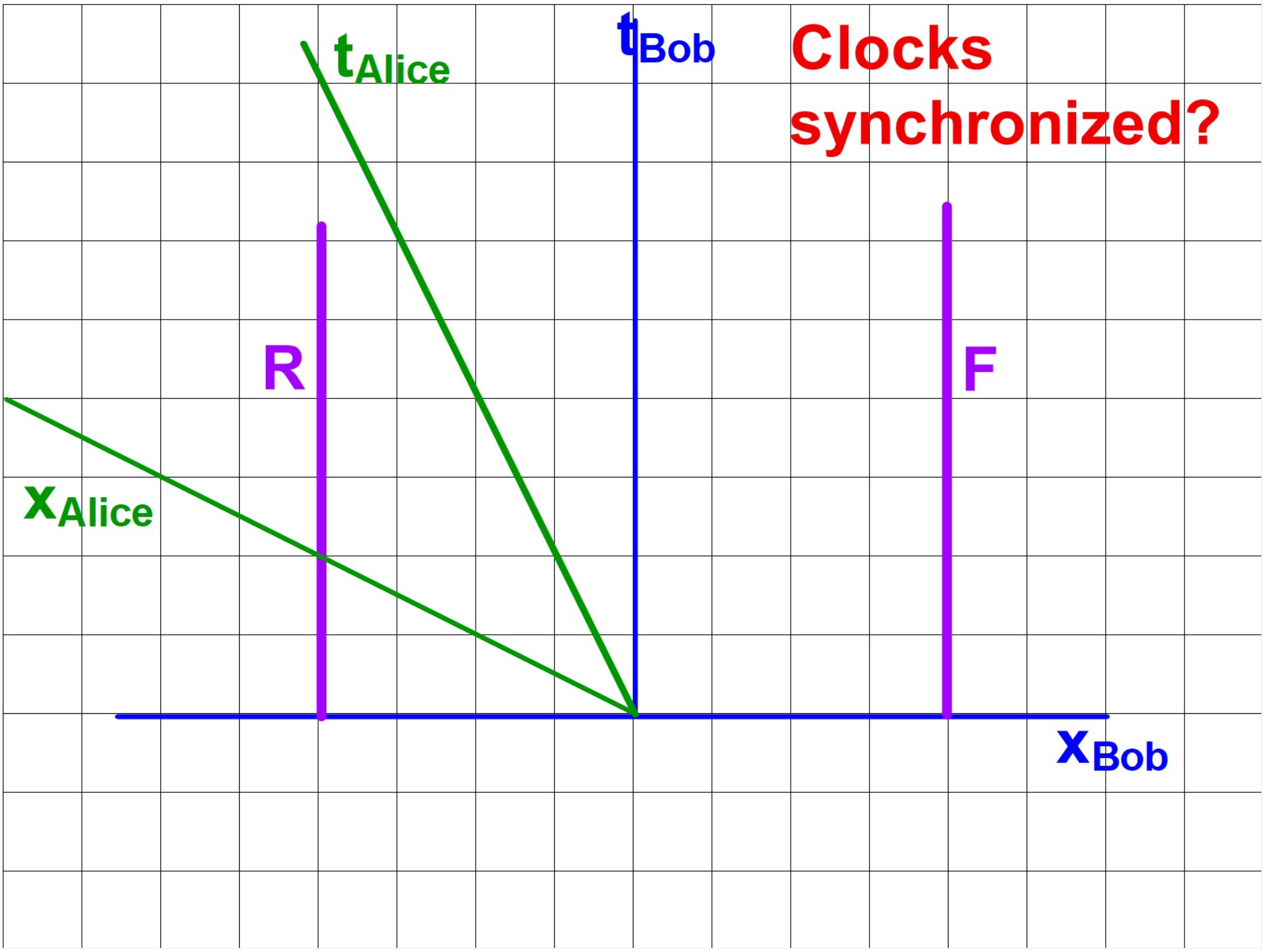
Are P & Q simultaneous?
t_train
t_ground
train says: NO
Q happens first
P
Q
x_train
lines of
simultaneity
x_ground


zoom!
R
F
Bob
Alice
Clocks
synchronized?

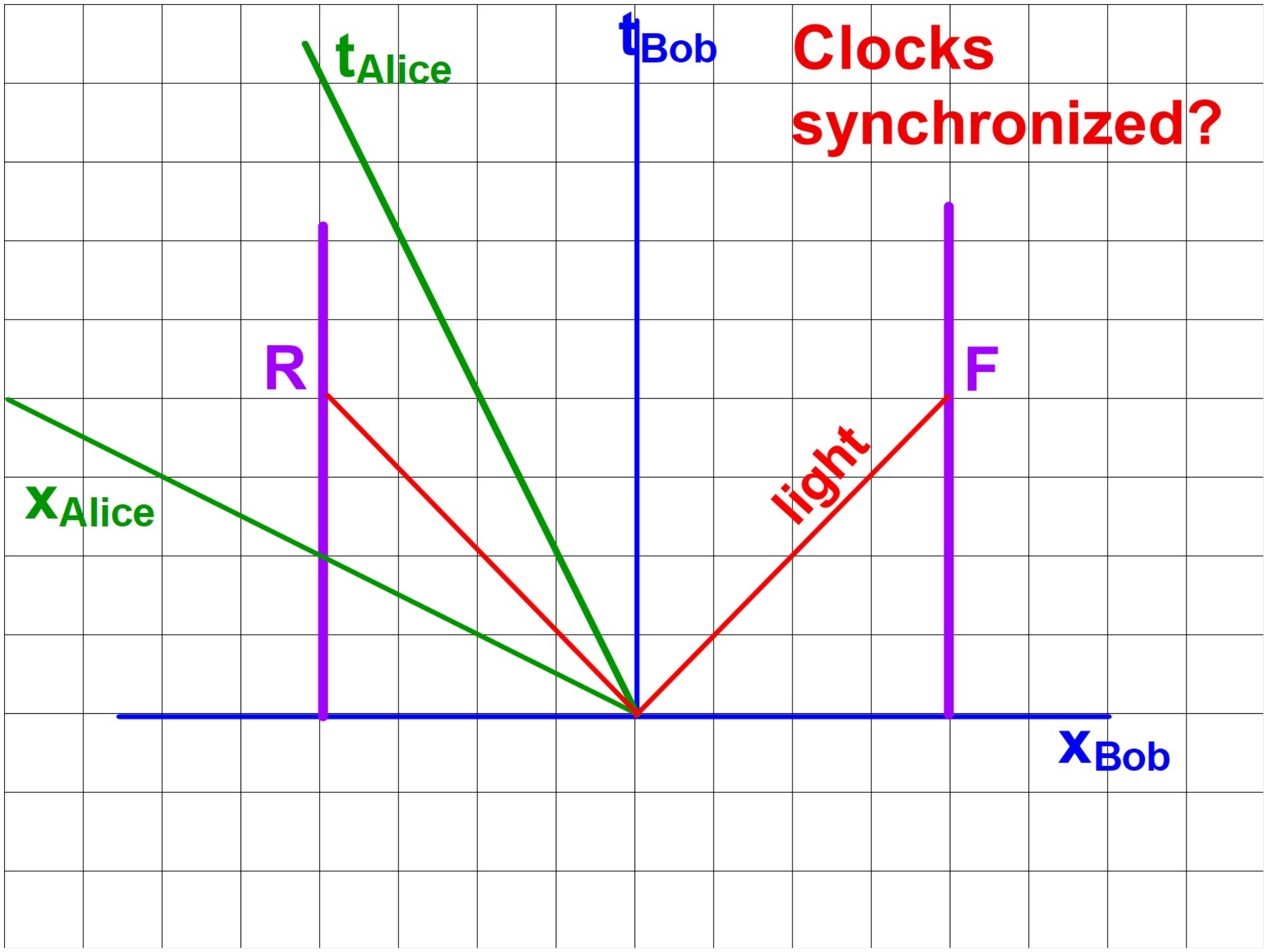

t
Clocks
synchronized?
Bob
R
F
x
Bob is at rest in Bob's frame.
The train walls are also at rest.



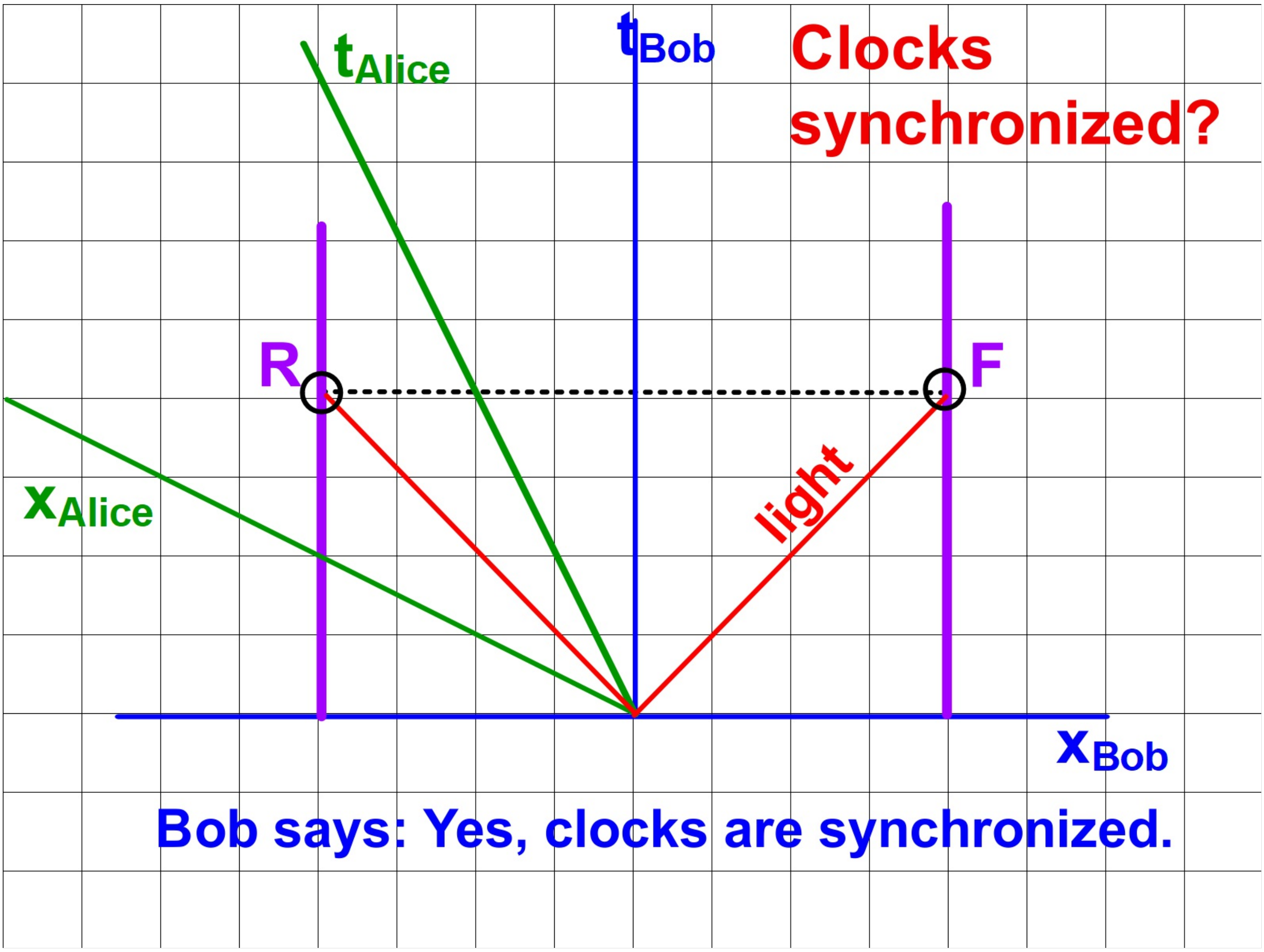
Bob is at rest.
Alice is moving to the left, with speed v.



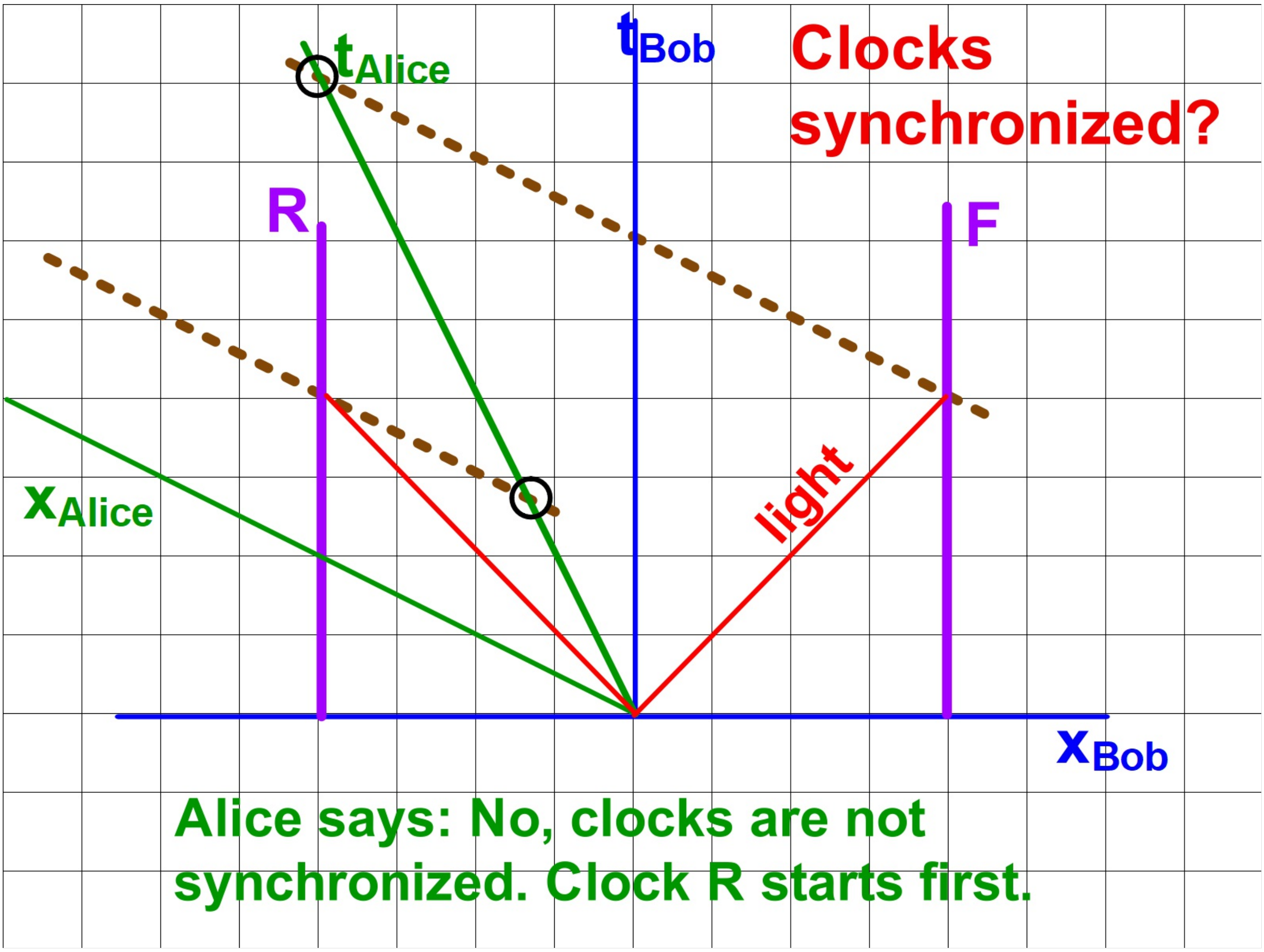
t_{Alice}
t_{Bob}
Clocks synchronized?
R
F
x_{Alice}
x_{Bob}



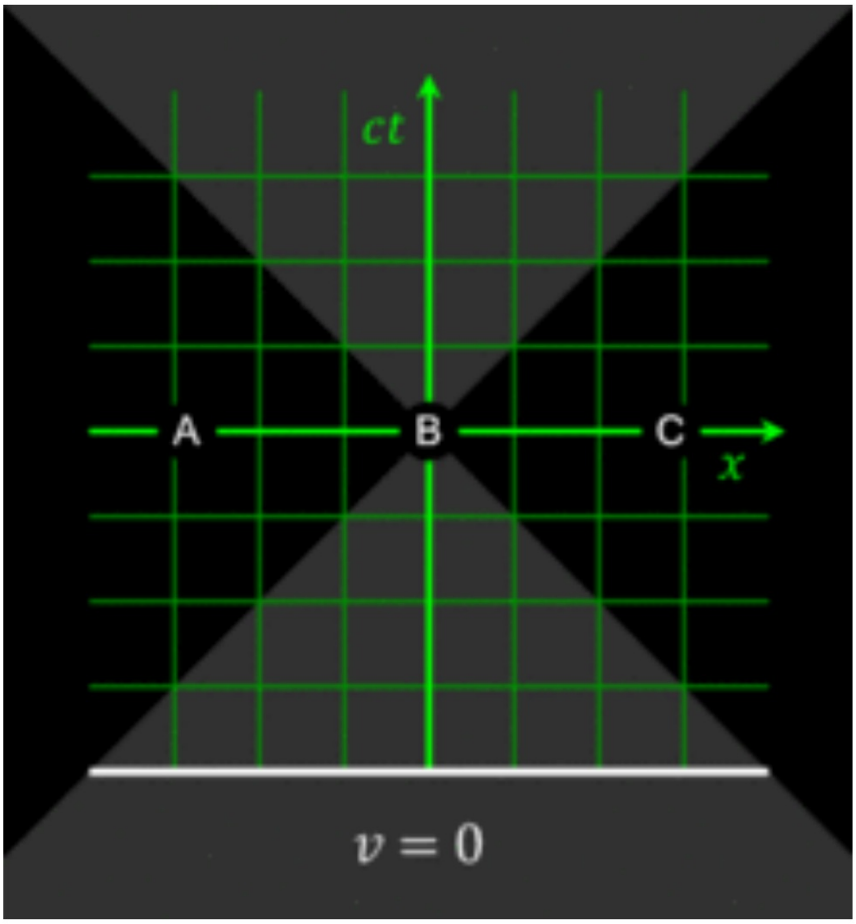
t_Alice
t_Bob
Clocks synchronized?
R
F
light
x_Alice
x_Bob



t_{Alice}
t_{Bob}
Clocks synchronized?
R
F
x_{Alice}
light
x_{Bob}
Bob says: Yes, clocks are synchronized.



Clocks synchronized?
t_{Alice}
t_{Bob}
R
F
x_{Alice}
light
x_{Bob}
Alice says: No, clocks are not synchronized. Clock R starts first.



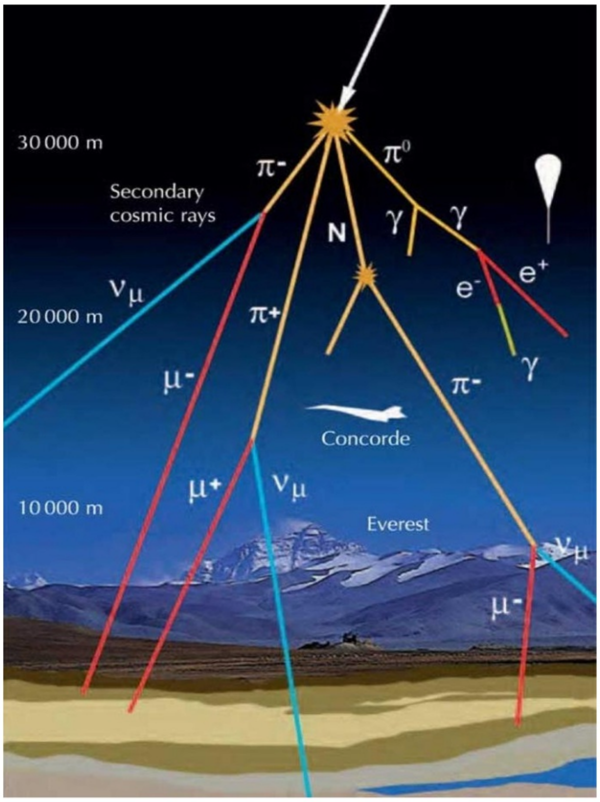
ct
A
B
C
x
$v = 0$

Muon Decay:
a real-world example of relativistic effects

cosmic rays create muons
in the earth's atmosphere

muons travel about 98% c

muons are detected
on the ground

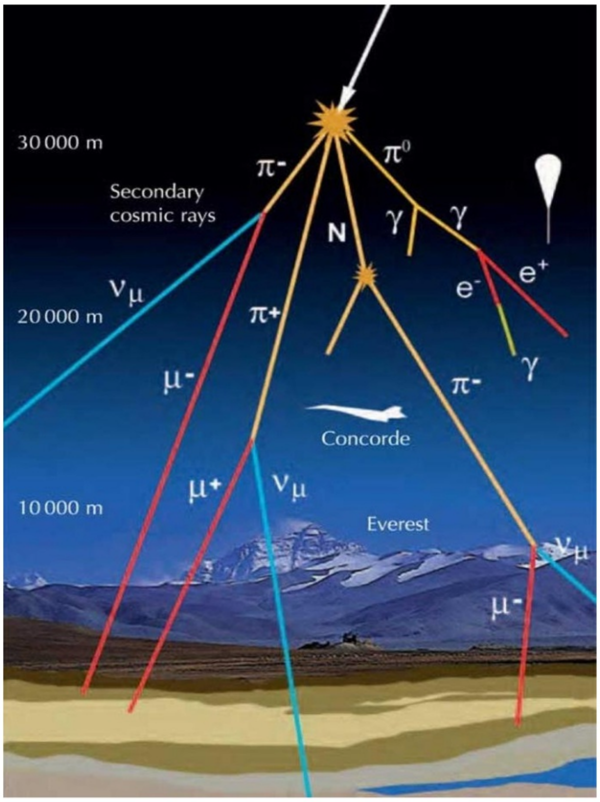


Muon Decay:
a real-world example of relativistic effects

muons travel about 98% c

muon lifetime ≈ 2 μs

How far can the muon travel in 2 μs? (non-relativistic)



Muon Decay:
a real-world example of relativistic effects

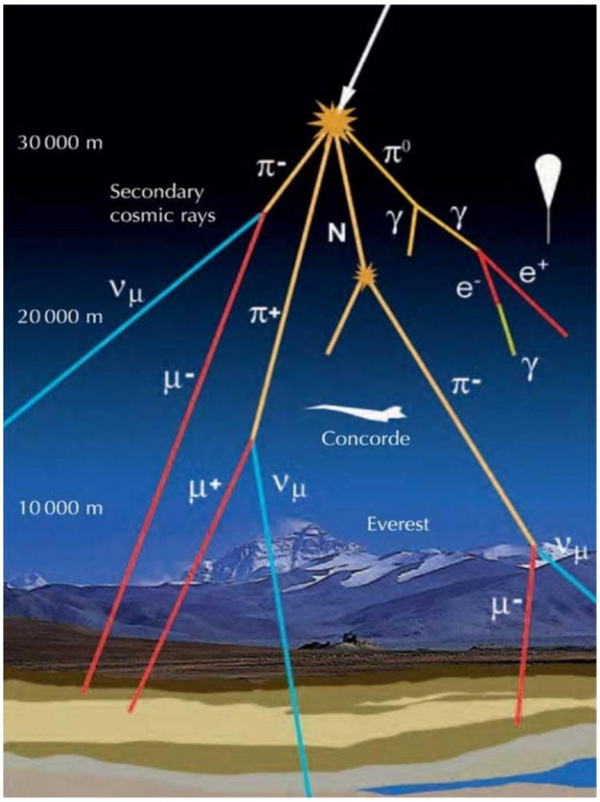
muons travel about 98% c

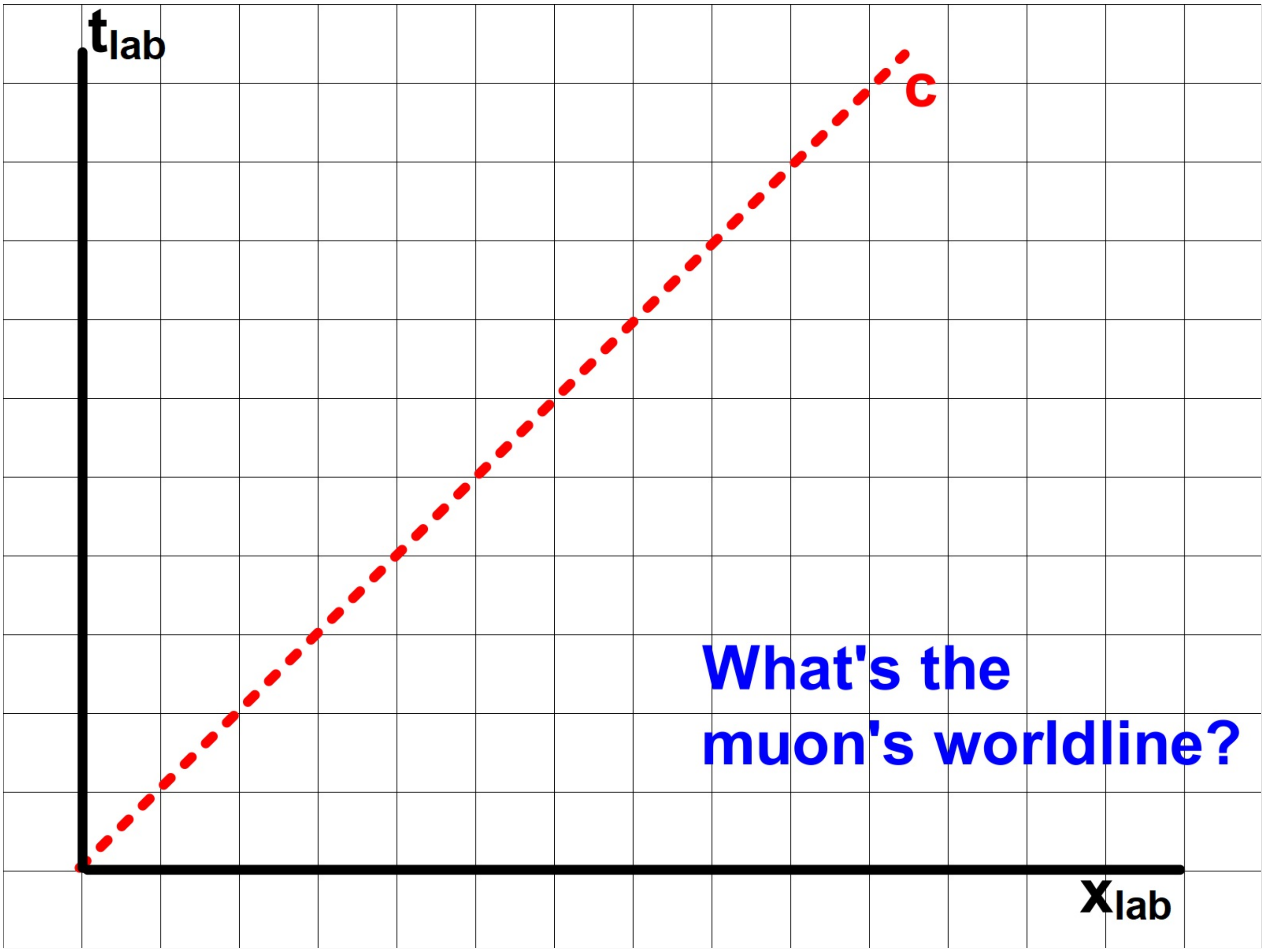
muon lifetime ≈ 2 μs

How far can the muon travel in 2 μs? (non-relativistic)

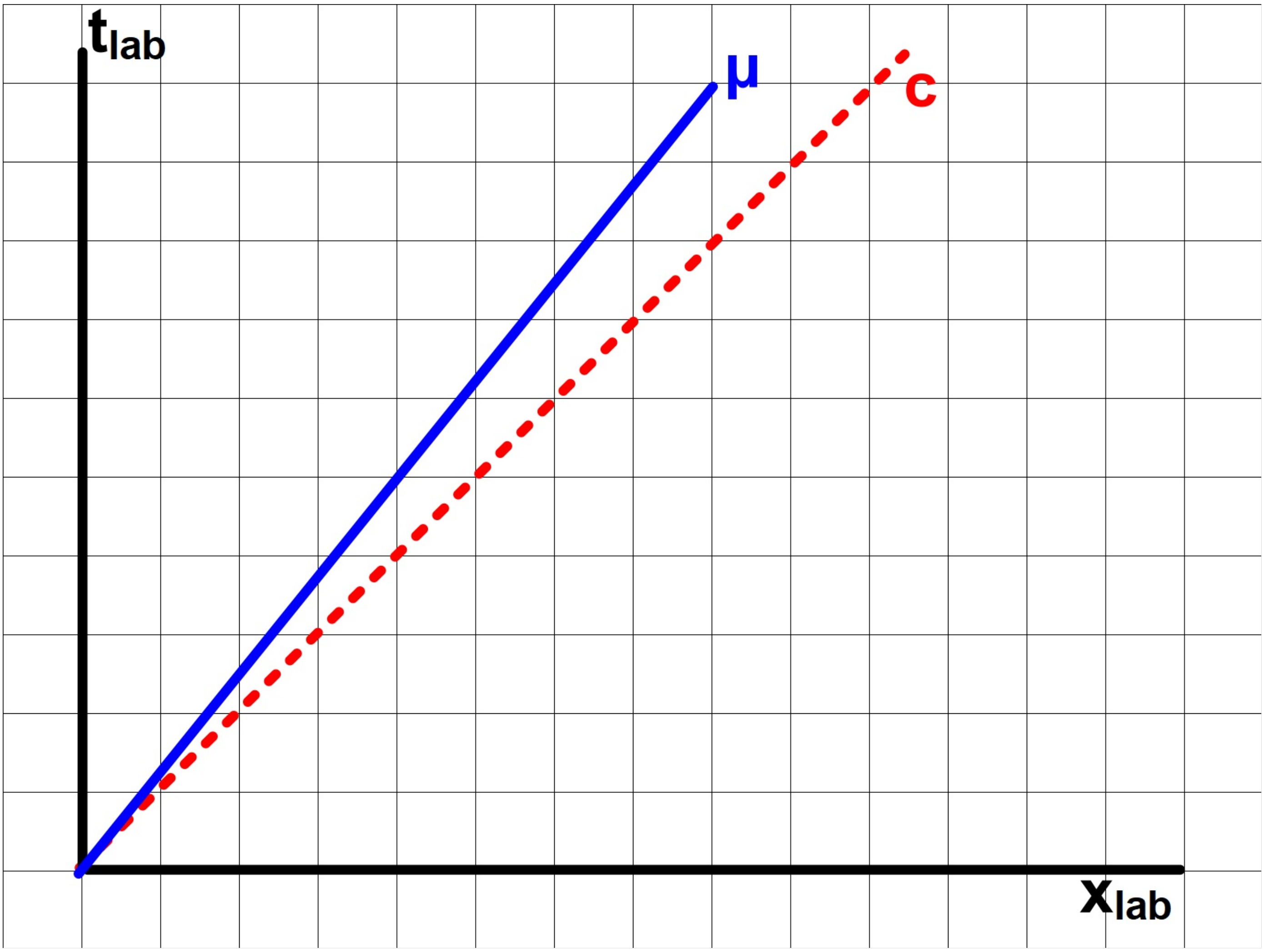
$\mathbf{v} \approx 3 \times 10^{8}$ m/s
$t \approx 2 \times 10^{-6}$ s

$\Delta x \approx 600$ m

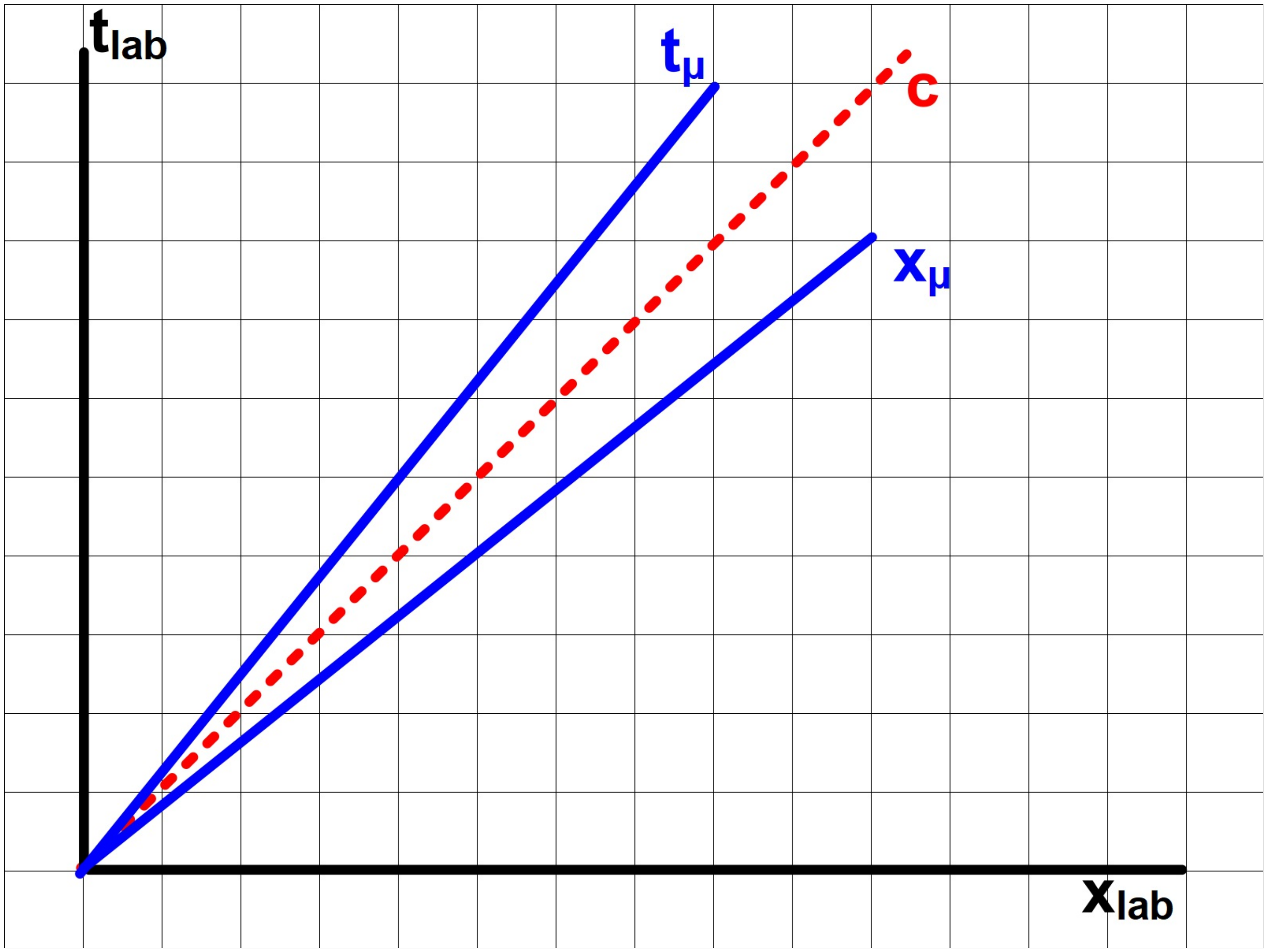




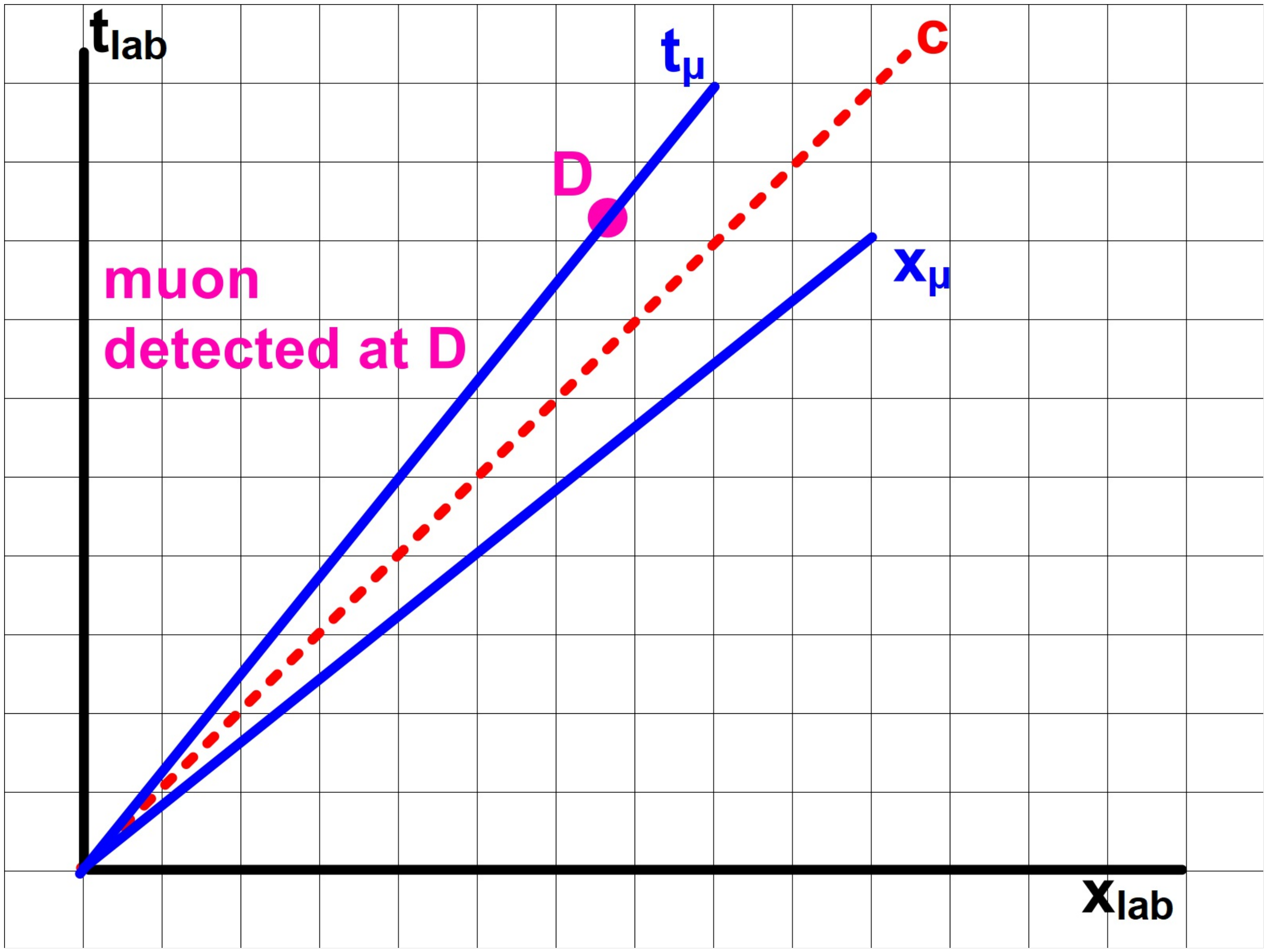
t_lab
c
x_lab
What's the
muon's worldline?



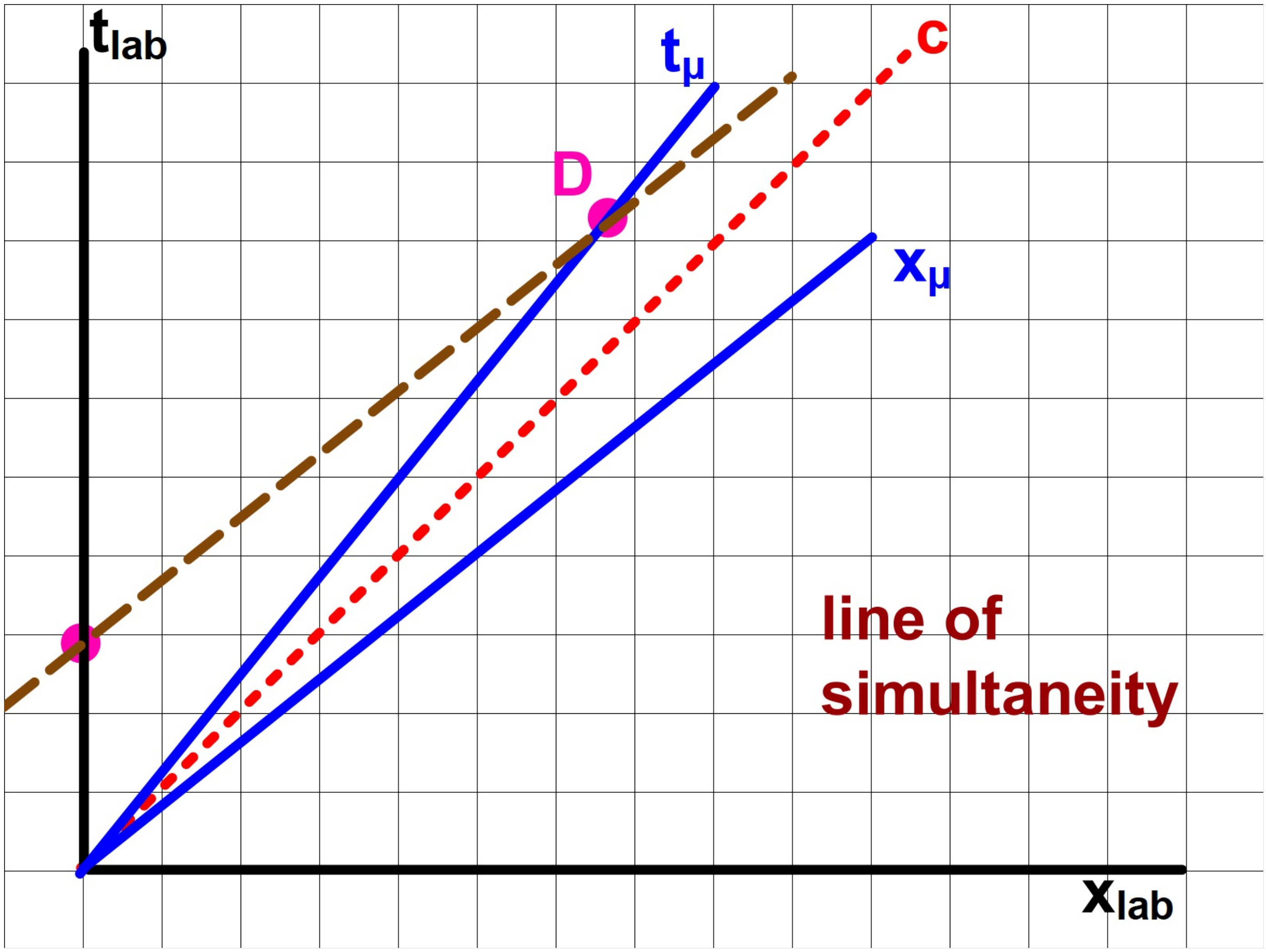
t_{lab}
μ
c
x_{lab}



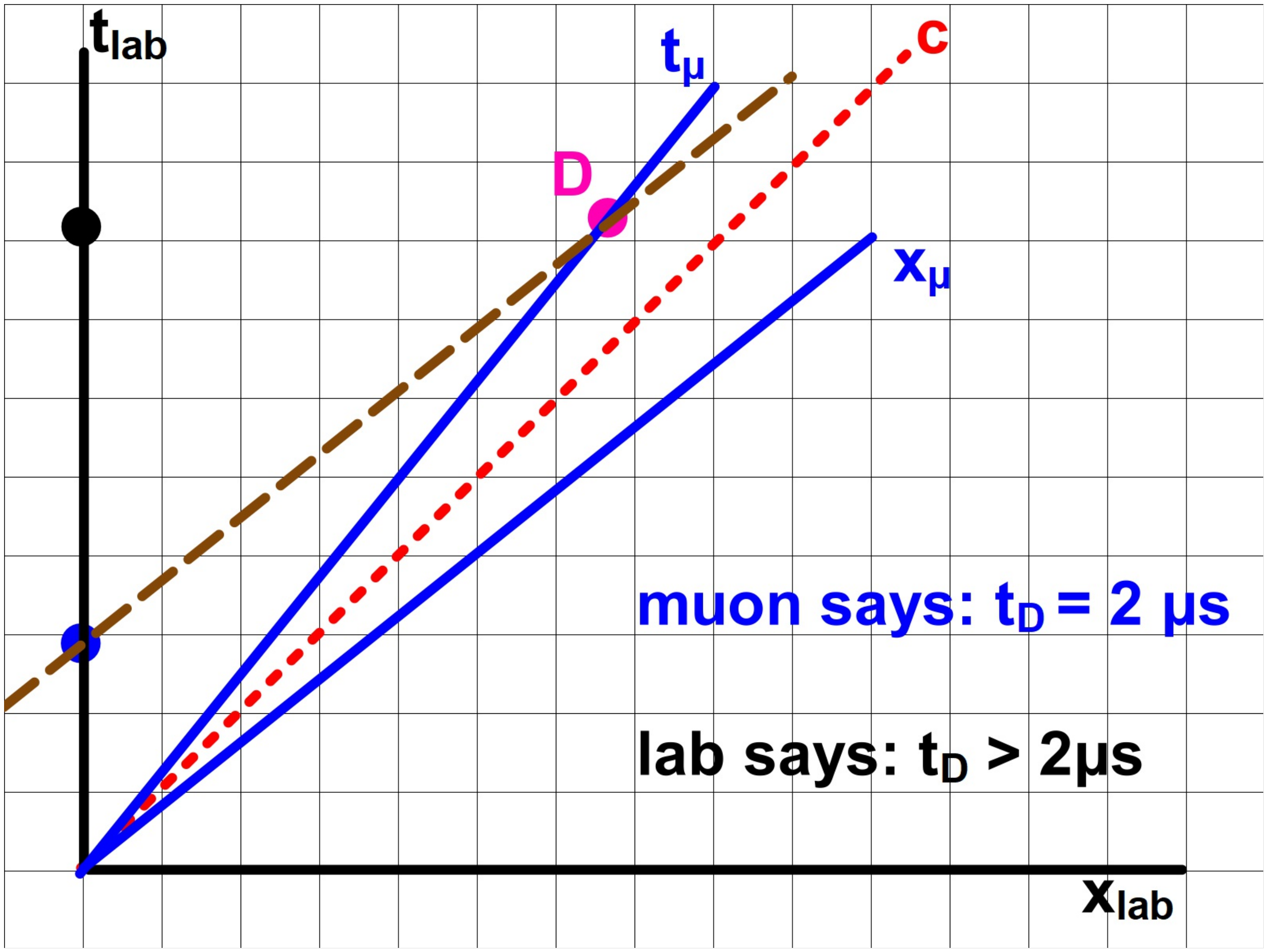
t_lab
t_μ
c
x_μ
x_lab



t_{lab}
t_μ
c
D
muon
detected at D
x_μ
x_{lab}



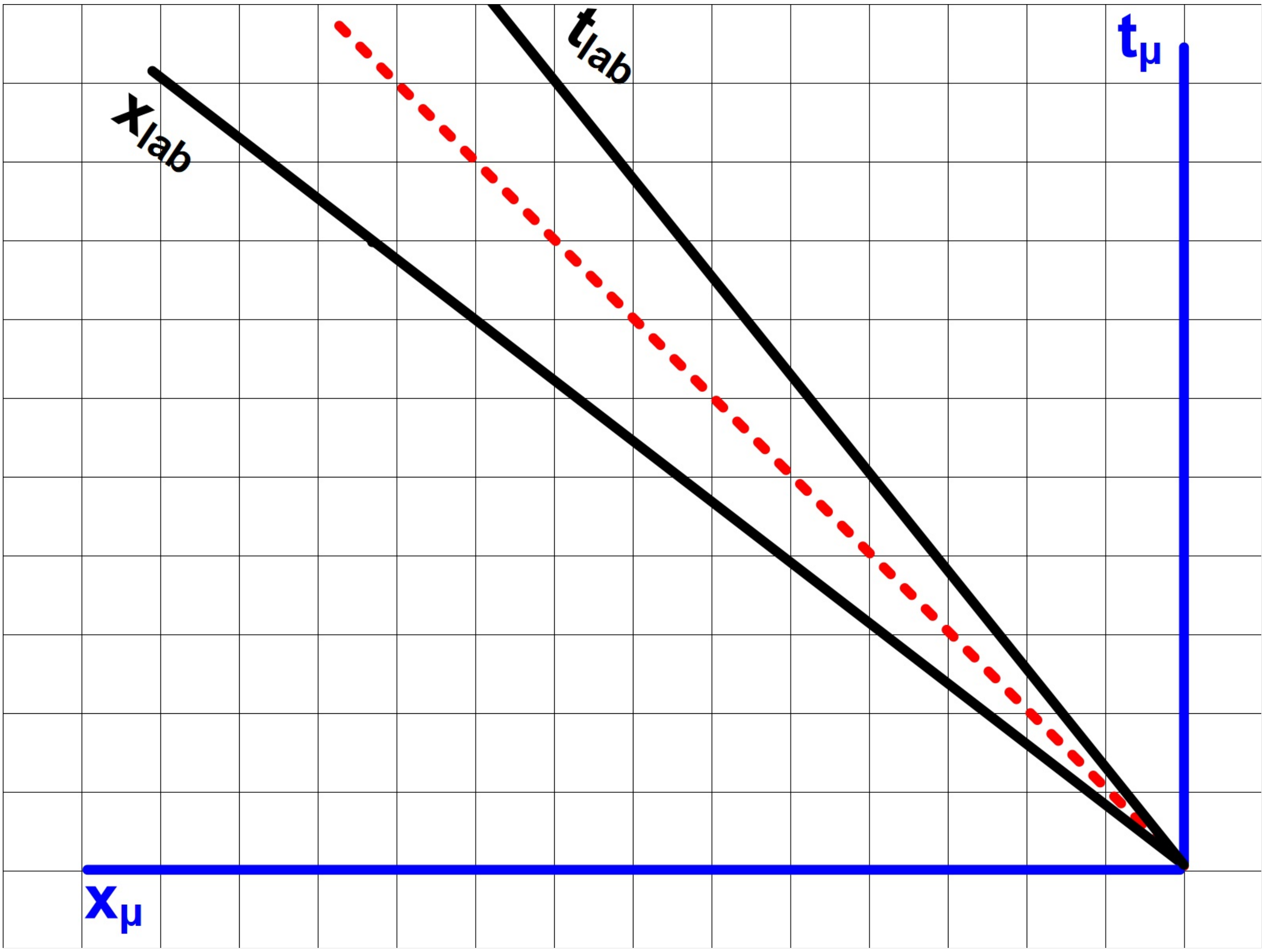
t lab
t μ
c
D
x μ
line of
simultaneity
x lab



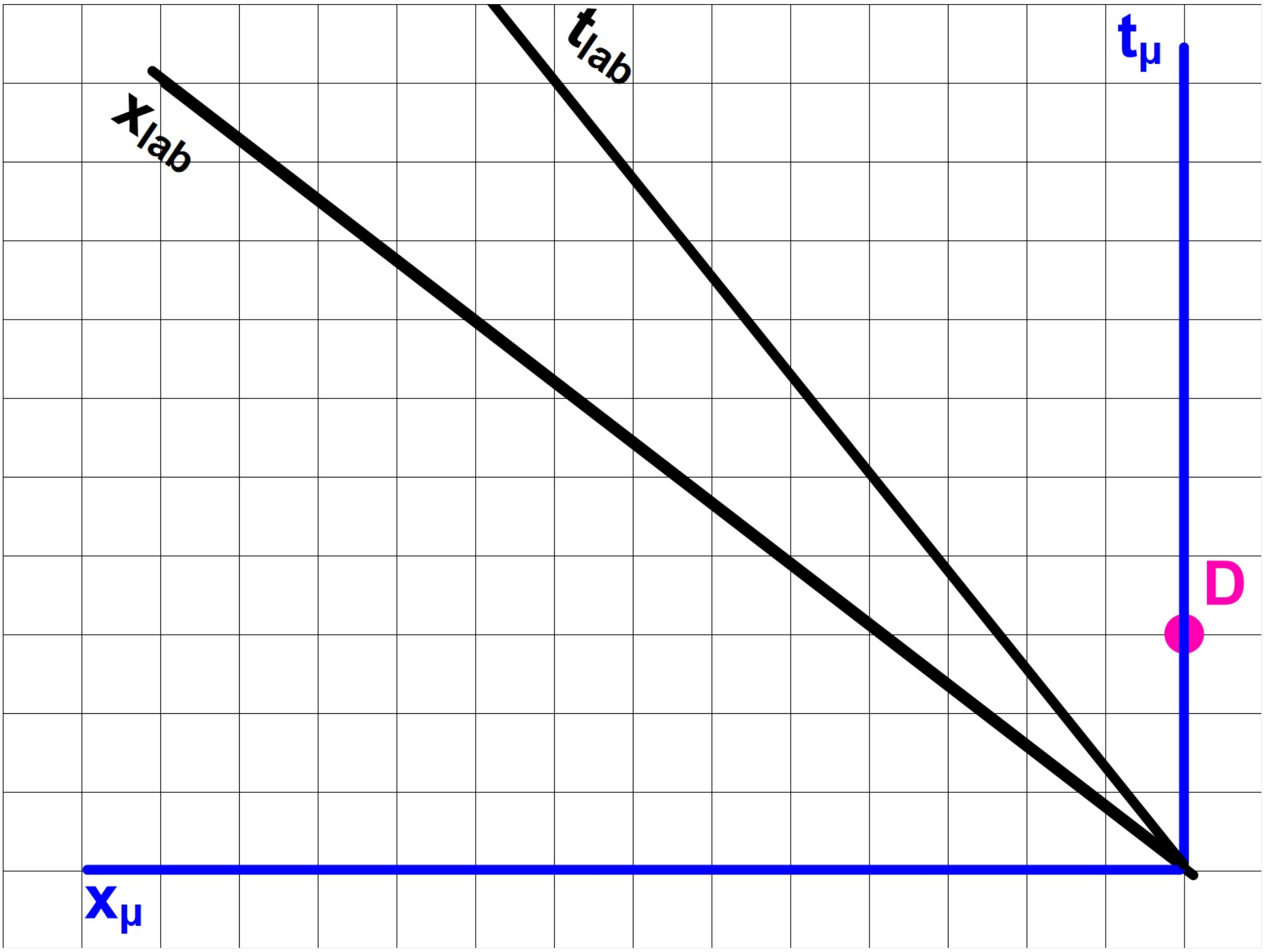
t_{lab}
t_μ
c
D
x_μ
muon says: $t_D = 2\ \mu s$
lab says: $t_D > 2\mu s$
x_{lab}

Equivalently, we can make the muon's worldline "vertical"

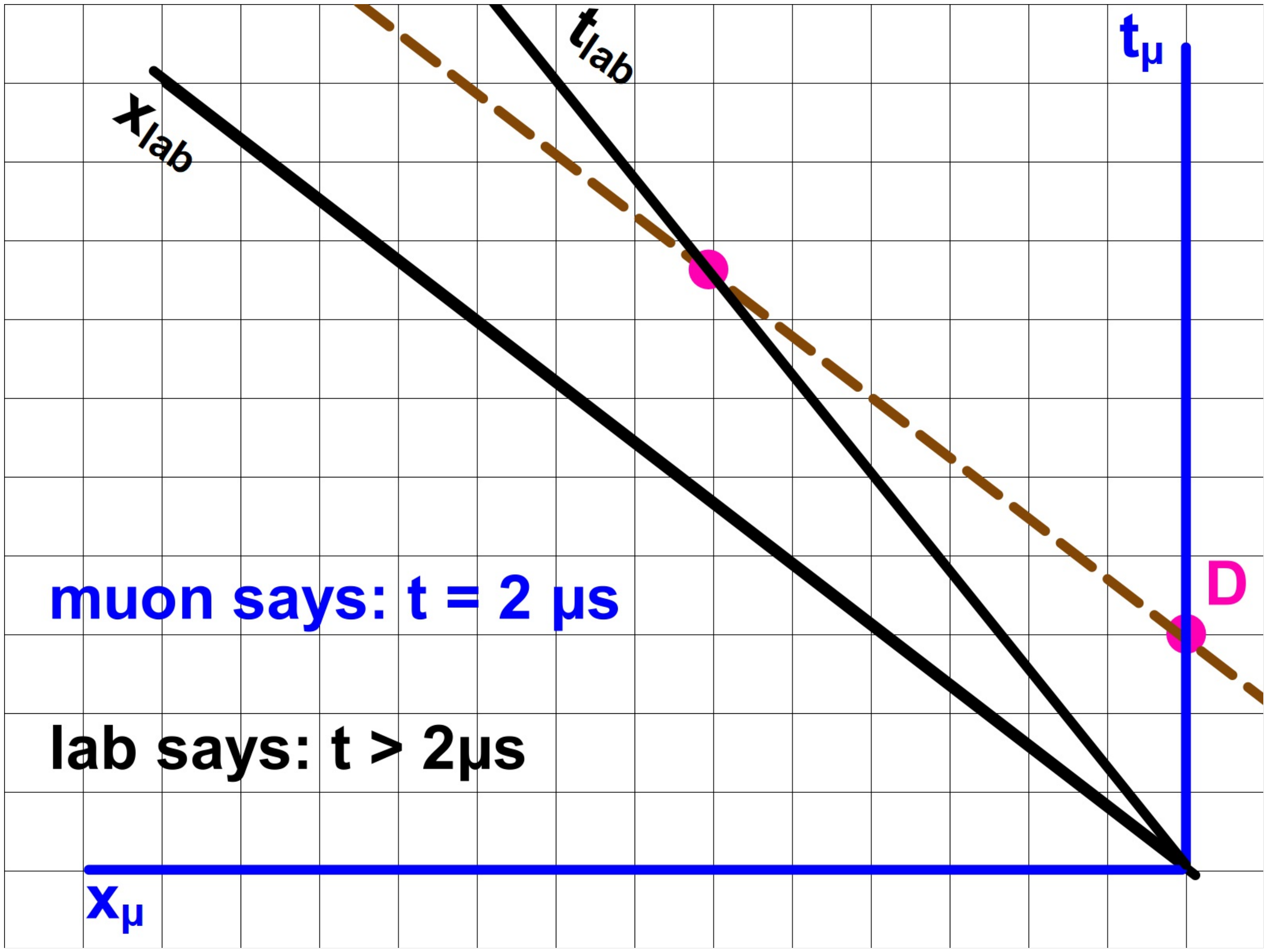
and the lab frame be the "squeezed" axes



t_{lab}
t_{μ}
x_{lab}
x_{μ}

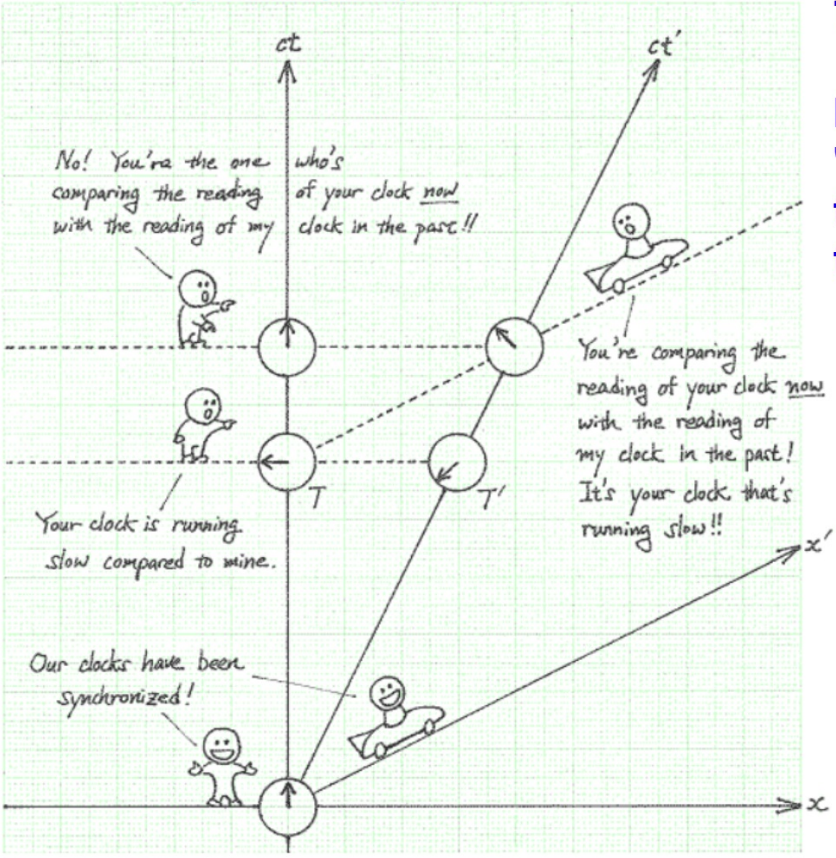


t_{lab}
t_μ
x_{lab}
D
x_μ



t_{lab}
x_{lab}
t_μ
D
muon says: t = 2 μs
lab says: t > 2μs
x_μ

Spacetime Diagram explanation of Time Dilation

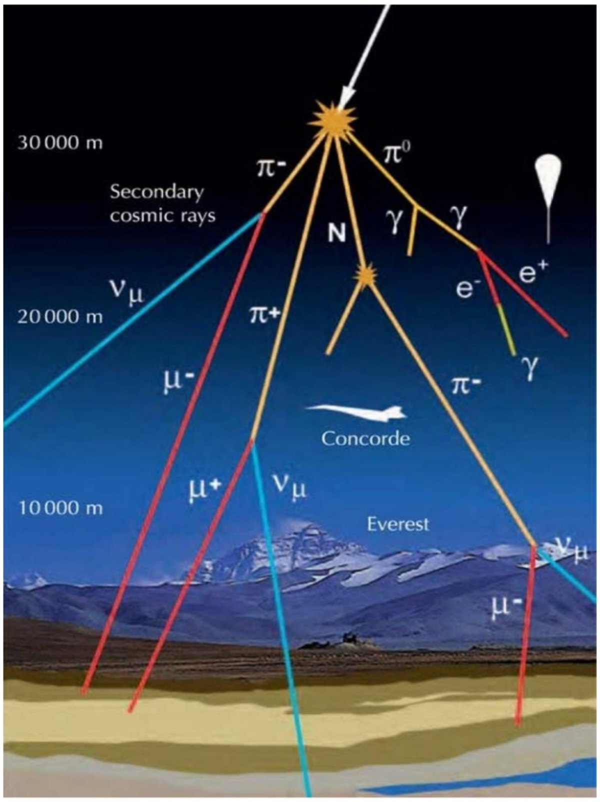


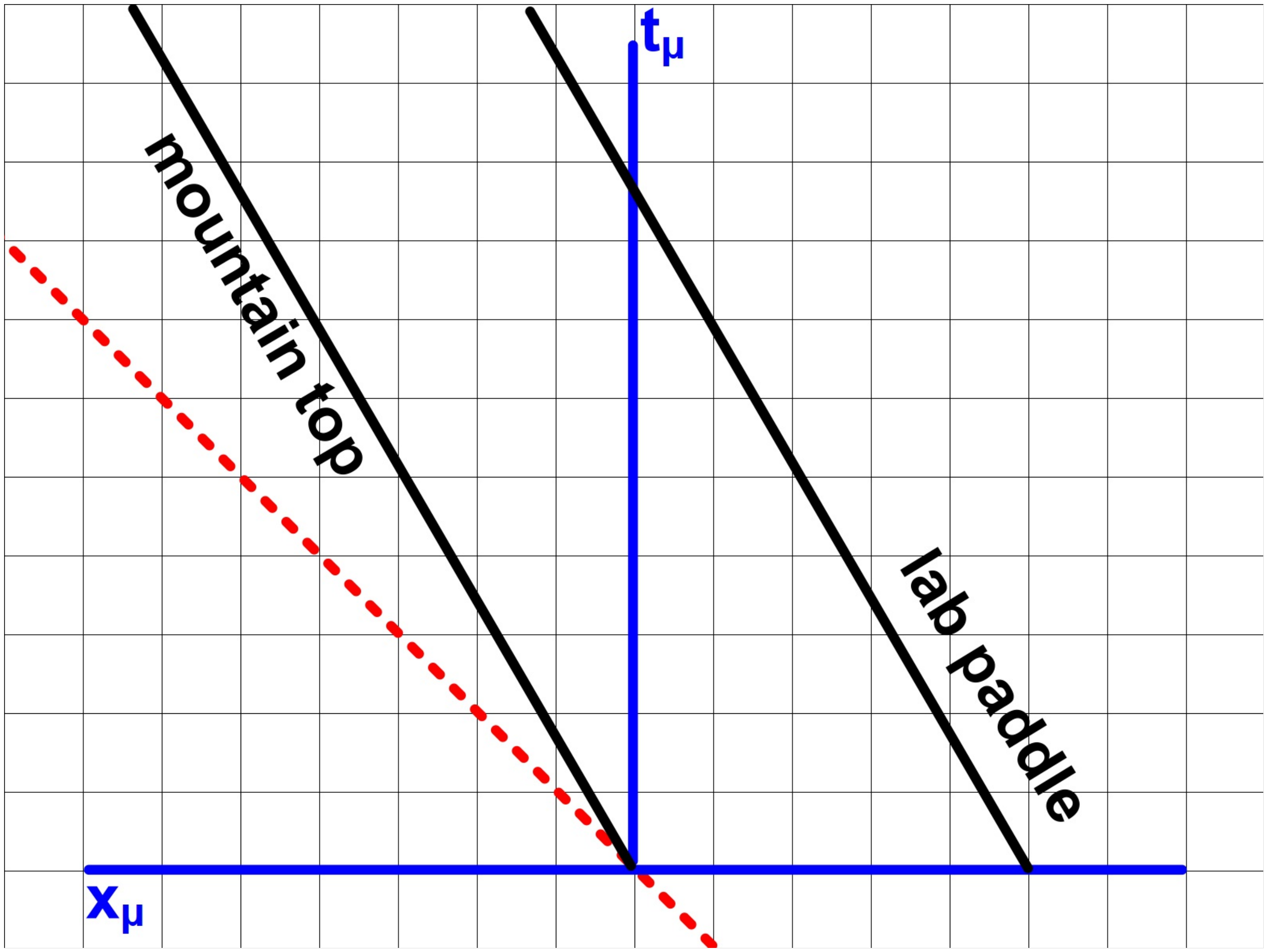
Time Dilation

Each frame "correctly" observes the other frame's clocks to be running too slowly.

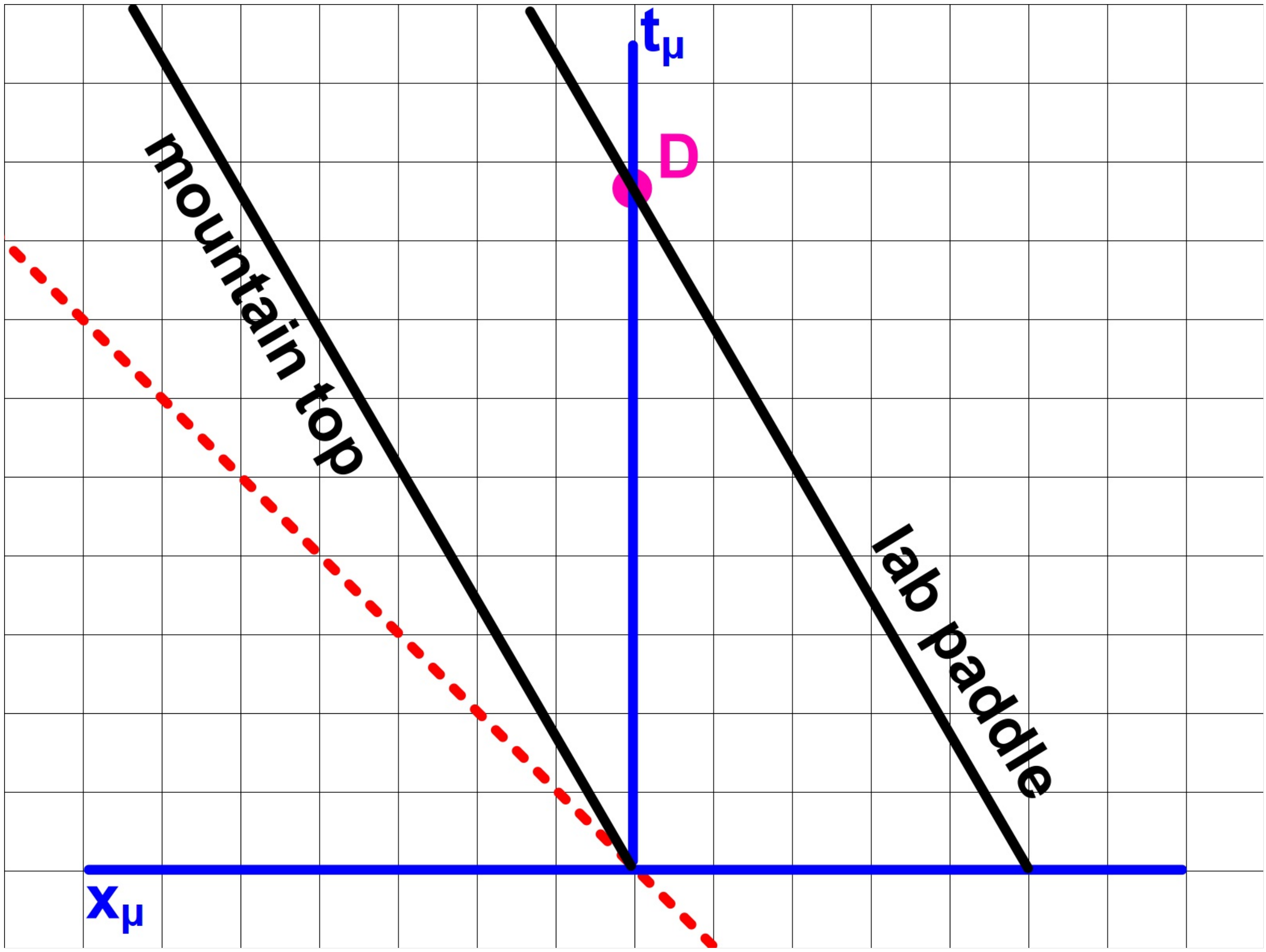
Muon Decay:
a real-world example of relativistic effects

How far does the muon travel?



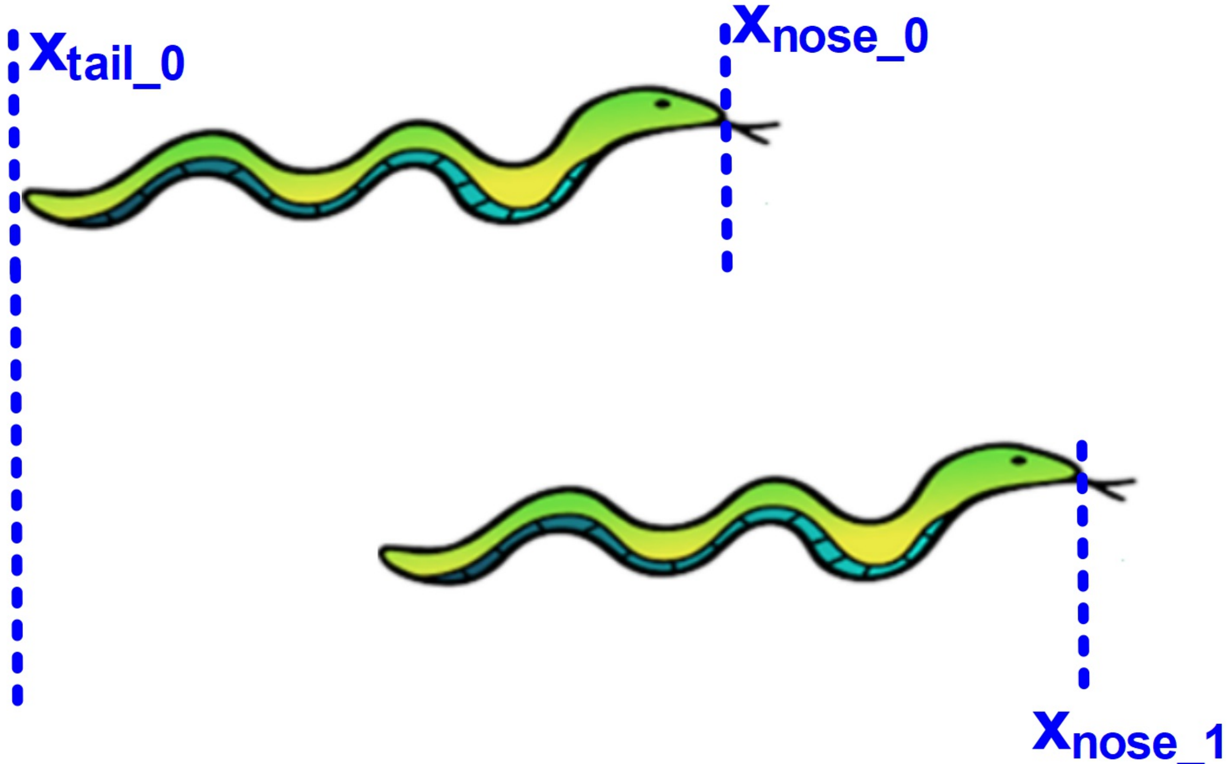


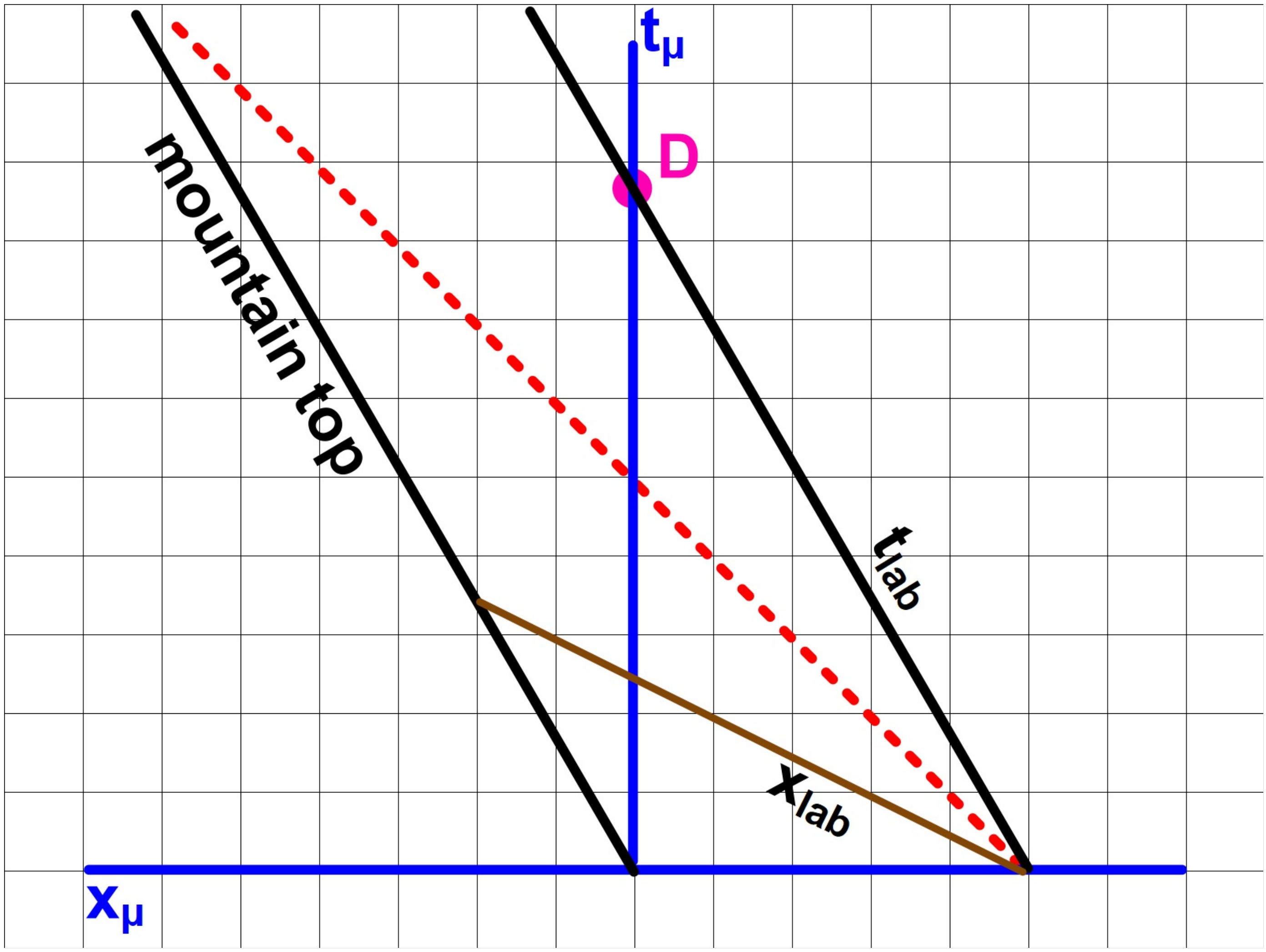
t_μ
mountain top
lab paddle
x_μ

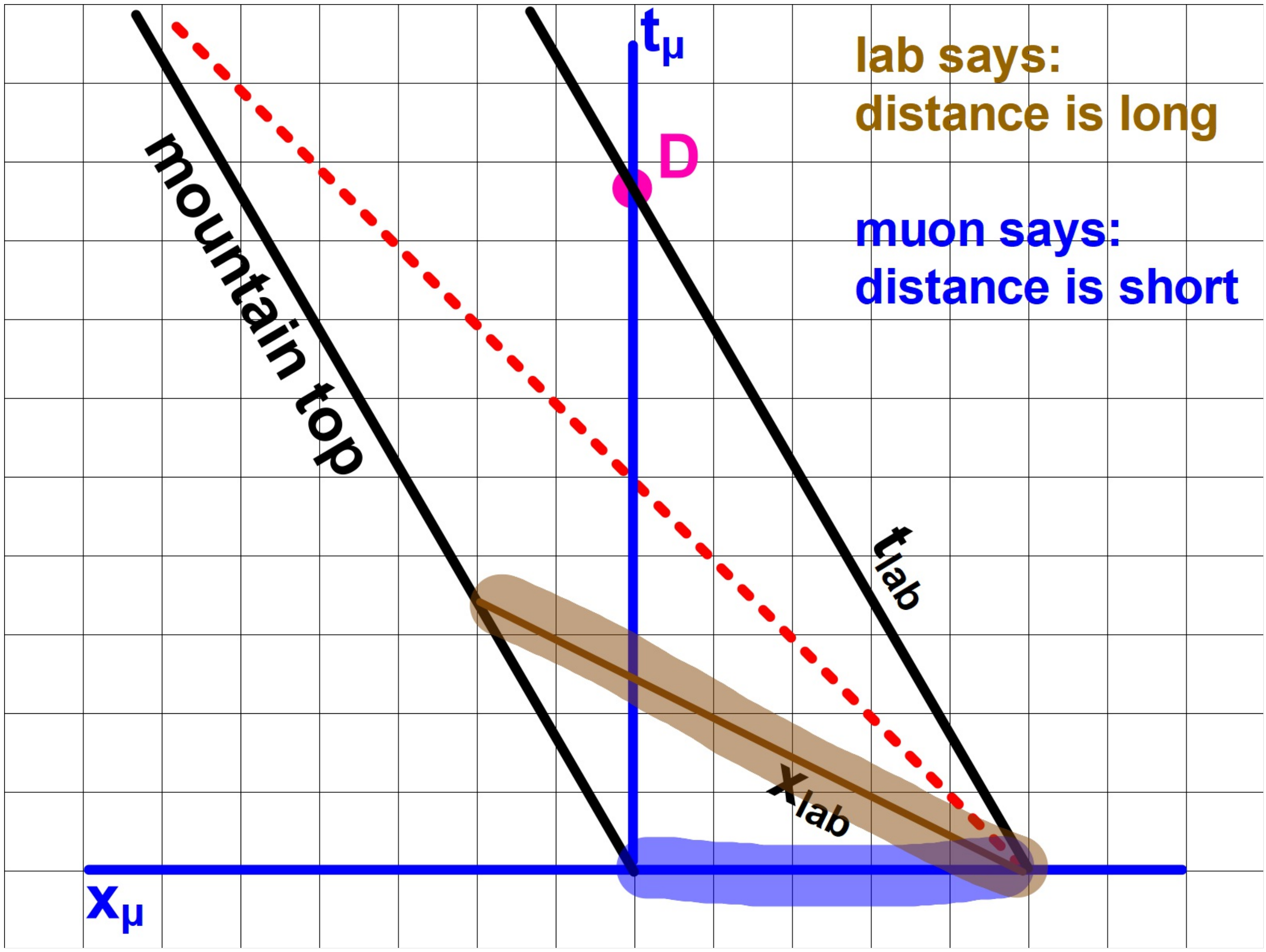


mountain top
lab paddle
D
t_μ
x_μ

What does the "length of something" mean?



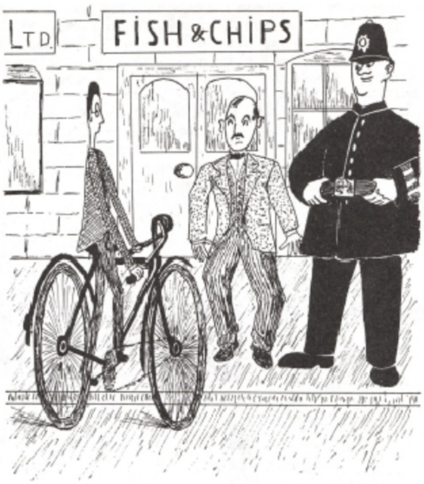

t_μ
D
mountain top
t_lab
x_lab
x_μ



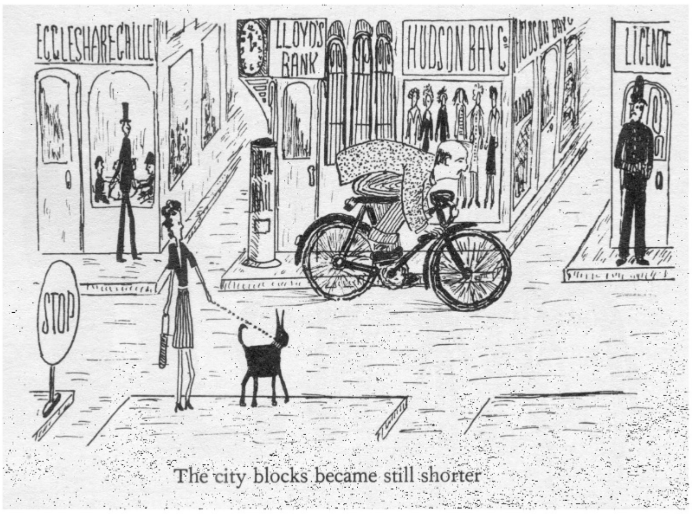
t_μ
lab says:
distance is long
D
muon says:
distance is short
mountain top
t_lab
x_lab
x_μ

Length contraction:

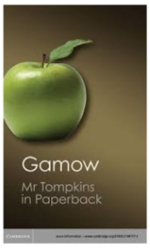
Moving objects have contracted lengths (in the direction of motion).



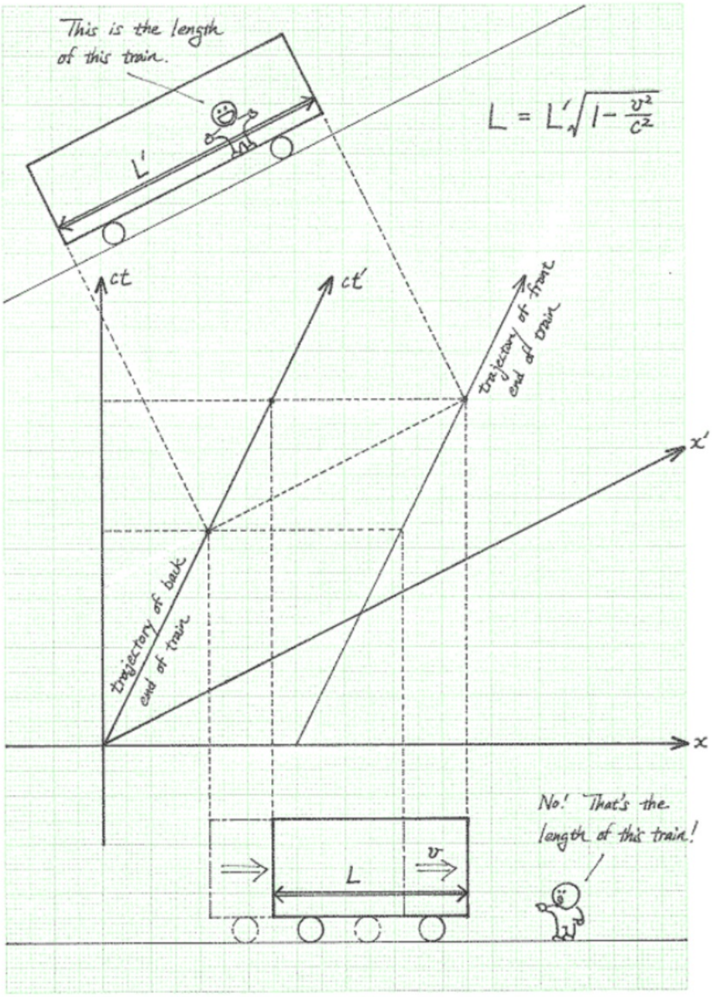
Unglaublich verkürzt



The city blocks became still shorter



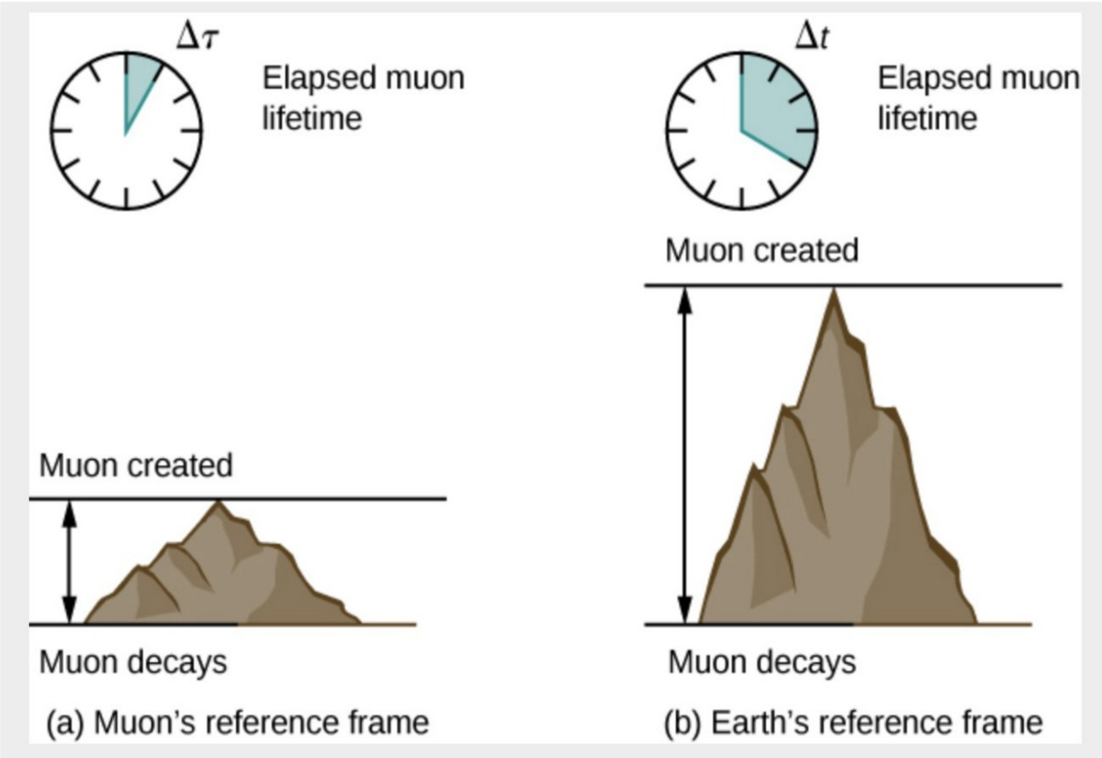
G.Gamow
Mr. Tompkins



Length contraction:

Moving objects have contracted lengths (in the direction of motion).

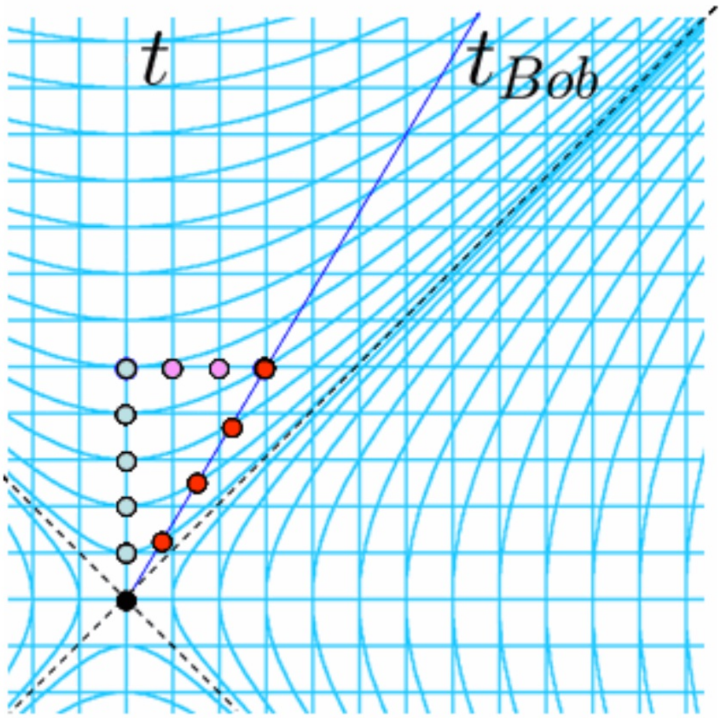
Muon Decay:
a real-world example of relativistic effects



(Or you can use hyperbolic graph paper)

Where the hyperbolas are asymptotic to v = +/- c.

Geometrically, a Lorentz transform is equivalent to sliding a spacetime coordinate along this hyperbola.



(I don't teach hyperbolic geometry to HS students though.)

Spacetime Diagrams

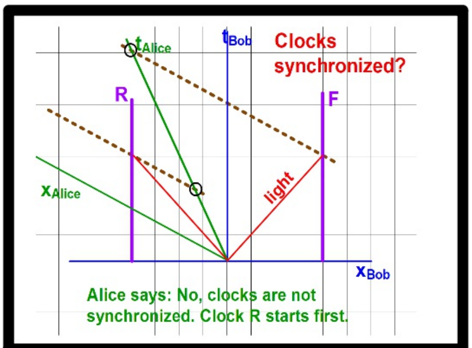
Input
- x vs. t graphs
- how to "read off" a graph
- x & t axes "squeezed"
  so that photon worldline
  "splits the uprights"

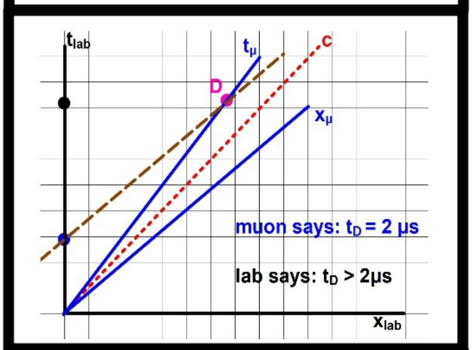
Output

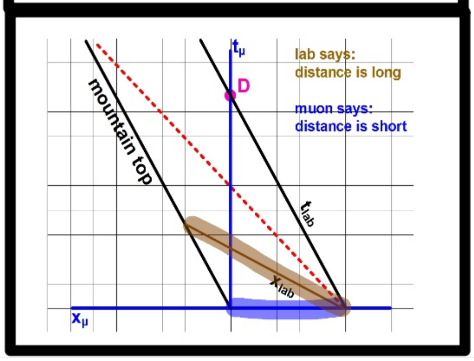
-relativity of simultaneity

-time dilation

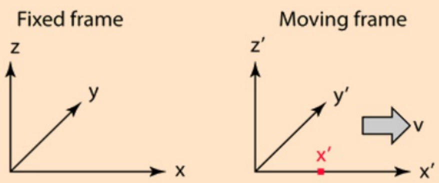
-length contraction







Lorentz Transformation



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

The reverse transformation is:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Much of the literature of relativity uses the symbols β and γ as defined here to simplify the writing of relativistic relationships.

Evaluation of symbols