MEAN LIFETIME PART 2: COSMIC MUONS
TEACHER NOTES

DESCRIPTION

Often physics students have experience with the concept of half-life from their lessons on nuclear decay. Teachers may introduce the concept using M&M candies as the decaying object. When students begin their study of decaying fundamental particles, their understanding of half-life may be shaky. The introduction of mean lifetime as used by particle physicists can cause more confusion.

Mean Lifetime Part 2: Cosmic Muons builds on *Mean Lifetime Part 1: Dice* which uses dice as a model for decaying particles. In this activity, students determine the mean lifetime of a fundamental particle found in the Standard Model using authentic cosmic ray data collected with a QuarkNet Cosmic Ray Muon Detector (detector). The students use plots generated by the Cosmic Ray e-Lab to determine the half-life and mean lifetime of cosmic muons using the same techniques used in *Mean Lifetime Part 1: Dice*. In addition, they have to consider the experimental conditions involved in measuring mean lifetime of fundamental particles.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

2. Developing and Using Models
3. Planning and Carrying Out Investigations
4. Analyzing and Interpreting Data
5. Using Mathematics and Computational Thinking
7. Engaging in Argument from Evidence
8. Obtaining, Evaluating, and Communicating Information

Crosscutting Concepts

1. Patterns.
2. Cause and effect: Mechanism and explanation.
3. Scale, proportion, and quantity.
4. Systems and system models.
7. Stability and change.

Common Core Literacy Standards

Reading

9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

IB Physics Standards

Topic 7.1

Understandings. Half-life.

Application. Investigation of half-life experimentally (or by simulation).

Utilization. Exponential functions.

Topic 7.3

Understandings. Quarks, leptons, and their antiparticles.

Topic 12.2 (AHL)

Understandings. The law of radioactive decay and the decay constant.

Applications and Skills. Solving problems involving the radioactive decay law for arbitrary time intervals.

ENDURING UNDERSTANDINGS

- Particles decay in a predictable way, but the time for any single particle to decay is probabilistic in nature.
- Scientists can use data to develop models based on patterns in the data.

LEARNING OBJECTIVES

Students will know and be able to:

- Using a decay curve, describe how half-life and mean lifetime can explain how particles decay randomly yet decrease in number in a predictable way.
- Explain the difference in the mathematical models used to determine half-life and mean lifetime.
- Determine the half-life and mean lifetime using a decay curve of a system of particles.
- Make a claim supported by evidence for the choice of mean lifetime to describe particle decay.
- Provide evidence to refute the claim that "All particles of a particular type decay in exactly a time described by the particle mean lifetime."
- Explain why the step of subtracting the background is important in determining the lifetime from an exponential decay curve.
- Explain why defining an N_0 is necessary for plots from the Cosmic Ray e-Lab.
- Describe how the results for mean lifetime of a cosmic muon change if a different N_0 is chosen.
- Describe how the results for mean lifetime of a cosmic muon change if the background is chosen at a value that is either too low or too high.

PRIOR KNOWLEDGE

Students must be able to

- Keep careful records of observations and add integers.
- Make and interpret graphs.
- Understand exponential functions.
- Distinguish between a curve which is exponential, a power of e, and a curve that is quadratic, a power of x.

BACKGROUND MATERIAL

When elementary particles decay into daughter particles, each particle takes a different amount of time to decay. The process is governed by probability such that different kinds of particles have different probable rates of decay. For example, a π^+ or π^- meson might have a mean lifetime on the order of tens of nanoseconds while a muon might have a mean lifetime in the microsecond range. This means that in the case of the π^+ meson, an initial sample N_0 will reduce to N_0/e after one mean lifetime, N_0/e^2 after two mean lifetimes, etc. For any one of these mesons, we cannot predict when it will decay; we can only predict the most likely time it will take to decay.

We can probabilistically predict the decay behavior and the typical mean lifetime of each type of "particle" using the analysis of an exponential decay curve. We use muons in this activity. The detector measures muon decay one muon at a time.

- The *half-life* of the particle (not generally used by particle physicists but useful to compare with radioactive half-life) is the time for ½ the sample to decay according to the mathematical model

$$N = N_0 2^{-t/T_{1/2}}$$

where N is the number of muons in the sample, N_0 is the initial number of muons, t is time, and $T_{1/2}$ is the half-life.

- The *mean lifetime* of the muon is the time for 1/e of the sample to decay according to the mathematical model

$$N = N_0(e^{-t/\tau})$$

where N is the number of muons in the sample, N_0 is the initial number of muons, t is time, and τ is the mean lifetime.

The mean lifetime is defined as the time for a sample size of N_0 cosmic muons to decay to a sample size of N_0/e cosmic muons; it takes two mean lifetimes to decay down to a sample size of N_0/e^2 cosmic muons, etc.

The mathematical model for half-life and mean lifetime must describe the same exponential decay curve. For this to be true the following equality must hold:

$$N_o 2^{-t/T_{1/2}} = N_o e^{-t/\tau}$$

Notice that N_0 cancels. Now operate on both sides with ln().

$$\frac{-t}{T_{1/2}}\ln(2) = \frac{-t}{\tau}$$

The equation relating mean lifetime to half-life simplifies to

$$\tau = \frac{T_{\frac{1}{2}}}{\ln(2)}.$$

We can include particles that appear and decay at different times in the same sample size, N_0, as long as we keep careful track of the time to decay for each particle.

In a detector, muon lifetimes are measured as the decay time for muons stopped in the detector:

- A low-energy muon enters a counter causing an initial scintillation and stops.
- The computer records time t_1 of resulting signal.
- After some time, the muon decays into an electron and two neutrinos resulting in a second scintillation.
- The computer records time t_2 of resulting signal.
- The computer records the time difference $t = t_2 - t_1$ as a line in the data file; this is the decay time for this particular muon.
- The process is repeated for many more low-energy muons; each has its own value of t.
- A histogram of values of t shows time on the horizontal axis and the number of remaining muons not decayed on the vertical axis.

IMPLEMENTATION

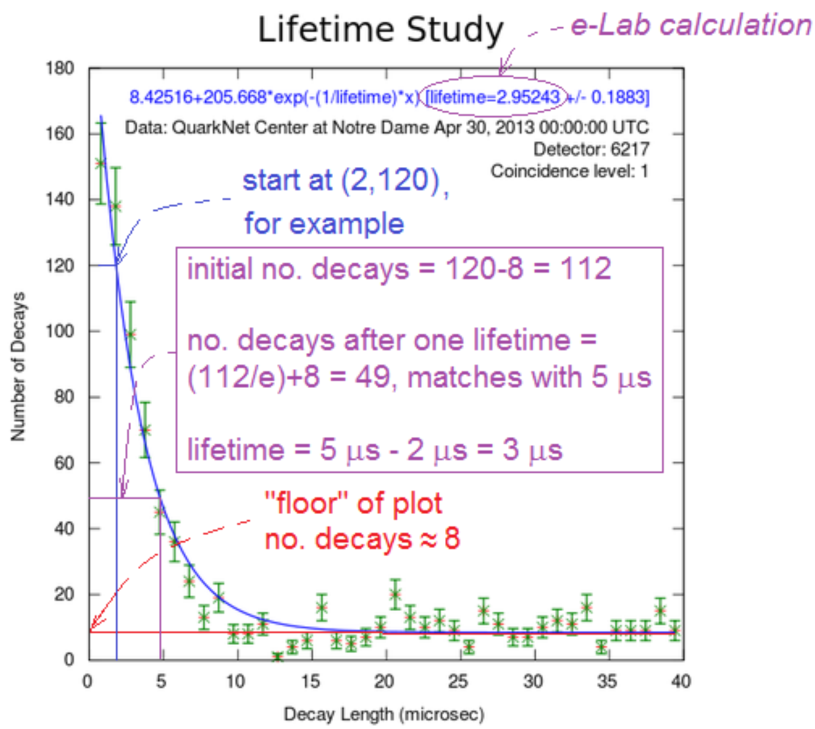
Give your students muon decay plots generated from detector data stored in the Cosmic Ray e-Lab. Stress to students that these are authentic experimental data of muon decays. The plots are in a separate document. Students do much the same work in finding half-life and mean lifetime as they did in the activity *Mean Lifetime Part 1: Dice* with the following additions:

- The actual muon lifetime plots do not start with t=0, so students must pick a point on the curve and re-calibrate that to t=0.
- There are background events that look like a muon decay (two distinct scintillations in one counter) but are not real decays. These are accidental combinations of noise in the photomultiplier tubes or single muons. This background is fairly constant.
- As a result, the decay curve follows a function more like

$$N = N_b + N_0 e^{-t/\tau}$$

where N_b as the background level of events, t is time, and τ is the mean lifetime

A sample analysis of the e-Lab calculation of muon lifetime follows. The student pages give a complete explanation of the analysis steps.



The student page has this plot without the solution to use as a practice exercise. As they work, you can provide hints using the sample analysis:

1. Select a start time. In the example, t_1 = 2 microsec.
2. Write the ordered pair for that time: (2 microsec, 120 decays)
3. Identify the background level. In the example, a background of 8 decays is the "floor" of the plot. You can think of this as the horizontal asymptote.
4. Determine the initial number of decays: 120 decays – 8 decays = 112 decays.
5. Determine the number remaining after one mean lifetime:

   112 decays/e + 8 decays = 49 decays
6. Reading the ordered pair from the graph yields (5 microsec, 49 decays).
7. Calculate the mean lifetime:

   $t = t_2 - t_1$ = 5 microsec – 2 microsec = 3 microsec

The analysis for half-life is similar. The difference is that in step 5 above, t_2 is found by

112 decays/2 + 8 decays = 64 decays

Ordered pair: (4 microsec, 64 decays)

Half-life = 4 microsec – 2 microsec = 2 microsec

Place your students in groups of two or three. Make multiple copies of each plot. Having multiple groups analyze the same plots leads to a discussion of experimental uncertainty.

Each group can analyze one to three plots. Make a table on the board for each group to record their results for half-life and mean lifetime.

Half-life	**Mean Lifetime**

Instruct the students to make two histograms of class data, one for half-life and one for mean lifetime. The following questions serve as discussion points for informal assessment, or they can be used for formal summative assessments.

Possible extensions for independent student work, with increasing levels of sophistication:

1. If there are multiple classes doing this activity, collect the data from all classes into a table and have students plot and analyze the data on their own.
2. Have students repeat the plot creation and analysis using a spreadsheet and exponential trendline with an equation like $N=N_0e^{-kt}$. The lifetime should be 1/k and the half-life should be (ln 2)/k. See Background Material above.
3. Have students make their own plot from classroom data by hand, make a best fit curve, and then mathematically derive the equation, the lifetime, and the half-life.

ASSESSMENT

Have students discuss the plots and analysis in small groups and then report. They should address these questions as well as their own:

- How well does each plot fit using two half-lives? With two lifetimes? What does this say about the reliability of the plot?
  - *The time needed for the sample size to drop by one half is the same for any chosen number of particles. The time needed to drop the sample size by 1/e is the same for any chosen number of particles. Students should cite evidence from their graph to provide evidence for their answer.*
- Distinguish between half-life and lifetime by using an exponential decay curve to find the half-life and lifetime of a particular type of particle.
  - *Given a new plot as an assessment item, the students can complete the analysis described above.*
- Explain why the step of subtracting the background is important in determining the lifetime from an exponential decay curve.
  - *If the analysis is done without subtracting the background, the resulting half-life or mean lifetime would be too long. If another group had a different background level, the half-life and mean lifetime would not agree with the results from the first group. The resulting histograms of results would have a wider spread and thus have more uncertainty.*

If your students did the activity *Mean Lifetime Part : Dice*, the following analysis is appropriate.

- Describe the characteristics of the plot using the dice model. Compare the characteristics with the plot using muon data from the Cosmic Ray Detector.
  - *The plots from the dice model are very similar to the plots from the muon data. They both represent exponential data. The big difference is that with the muon data, an N_0 must be chosen and the background must be subtracted.*
- Are the values for half-life and lifetime using the dice model the same as the values found using Cosmic Ray Detector data? How are they similar and why?
  - *The plots are similar, but the values for half-life and mean lifetime are different.*

Students can also do one of the extensions for evaluation by the teacher. These extensions can be evaluated using the questions listed above.