Shut Up and Calculate: The Many Worlds of Quantum Mechanics

Surjeet Rajendran

Department of Physics & Astronomy, The Johns Hopkins University, Baltimore, MD 21218, USA

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Abstract

The usual presentation of the postulates of quantum mechanics in textbooks introduces a variety of ad-hoc axioms to describe the phenomena of measurement. These axioms make little logical sense even though they accurately describe the empirical phenomenology of measurement. In this note, I will show that these ad-hoc axioms are unnecessary. There is really only one axiom of quantum mechanics - namely the Schrodinger equation. By supplementing this equation with the assumption that the Hamiltonians that exist in the world are local, I will derive the phenomenology of measurement such as the collapse of the wave-function, the emergence of probability from the deterministic Schrodinger equation and the absence of measurements of macroscopic superpositions. Further, by observing that macroscopic bodies are in coherent (or other suitably localized) states, I will also show how the Born rule emerges from this description.

1 Introduction

Every professional physicist "knows" quantum mechanics *i.e.* given a quantum mechanical problem, the physicist can, with the help of suitable stimulants, perform the requisite algebra and obtain the correct result for what an experiment should see when a certain measurement is made. Our textbooks, after all, do a wonderful job of drilling algebra into the undergraduate mind. This communal expertise ends with algebraic ability - significant number of physicists do not understand what measurement actually means or what it implies about the natural world. Astonishingly, even though the concept of measurement has been understood for well over 50 years, our textbooks do a poor job of conveying this understanding. Moreover, there is bizarre sociology in the field - there is active hostility towards wanting to answer this question, best summarized by the "shut up and calculate" mentality *i.e.* one should stop asking questions about what a fundamental physical process means and replace this curiosity with an unscientific robotic attitude where the physicist is reduced to an algebraic tool.

Strange as this attitude is, I actually agree with it. A complete physical theory, which quantum mechanics is, should be able to describe all of its features via computation instead of sliding into philosophical discourse. But, when one engages in "shut up and calculate", one should perform the full computation and not stop midway. What you are going to see is that when you do perform the full calculation, quantum mechanics does in fact describe measurement in a very simple and direct way, even though its consequences are bizarre.

I will begin in section 2 by first reviewing the standard postulates of quantum mechanics as described in textbooks. In this section, I will spend some time making fun of the standard description of measurement and how absurd it is for any one to seriously believe it. Following this exercise in mockery, in section 3, I will show how the phenomenology of measurement can be derived from the Schrodinger equation as long as one makes certain assumptions about the Hamiltonians that exist in our world. There is nothing original in these parts of this paper - the understanding that I am hoping to convey exists in the community. But, I have not come across a note where all of it has been written down. In section 4, I will conclude.

2 The Postulates of Quantum Mechanics

Any reasonable textbook[1] on quantum mechanics would teach you that the postulates of quantum mechanics are as follows:

1. The complete physical state of a system is an element $|\Psi\rangle$ of a Hilbert space.

2. The time evolution of $|\Psi\rangle$ is described by the Schrodinger equation $i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$ where H, the Hamiltonian, is a Hermitean operator.

What do these postulates imply? First, since the Hamiltonian H is Hermitean, the time evolution is unitary *i.e.* it is reversible. Second, the evolution is linear *i.e.* if there are a set of states $|\Psi_k\rangle$ that all individually obey the Schrodinger equation, then so does the sum $\sum_k c_k|\Psi_k\rangle$ for any arbitrary c_k. There is also no restriction on the physical size of the quantum state $|\Psi_k\rangle$. Protons, electrons, atoms, small molecules, proteins, *human beings* and planets all obey the Schrodinger equation. That is, all of these objects are capable of being placed in all sorts of superpositions. Importantly, the time evolution is fully deterministic - there is no probability in the Schrodinger equation. Given an initial condition, the equation exactly tells you what the final answer is without any uncertainty.

How well does the Schrodinger equation do in describing the world that we see?

Naively, not very well.

First, probability is one of the defining aspects of the phenomenology of quantum mechanics. How can probability come from a nobly deterministic equation like the Schrodinger equation? Second, unitary time evolution is reversible. But, the outcome of measurement in quantum mechanics is not reversible. Third, if macroscopic objects can be placed in superposition, then it should be possible to place human beings in superposition where the same human being is in multiple spatial locations. If that is the case, why do we not *see*[2] a macroscopic object in more than one location at the same time?

To explain all this, the textbooks invoke additional "measurement" postulates. These are:

1. The quantum state $|\Psi\rangle$ happily undergoes time evolution as per the Schrodinger equation. But then, a measurement of some operator O occurs. When this happens, the time evolution of the Schrodinger equation is temporarily halted.

2. The state $|\Psi\rangle$ gets expressed as $|\Psi\rangle = \sum_k \alpha_k|f_k\rangle$ where the $|f_k\rangle$ are eigenstates of the operator O with eigenvalue λ_k.

3. When the measurement is done, one "gets" the eigenvalue λ_k as the outcome of the measurement and the probability of getting λ_k is $|\alpha_k|^2$ (the so called "Born Rule").

4. No matter what the initial state $|\Psi\rangle$ is, the state after measurement "collapses" to $|f_k\rangle$.

5. This also explains why we do not see macroscopic superpositions where macroscopic objects such as human beings are in spatial superpositions since such macroscopic objects are always "being measured" and thus their quantum state always collapses to a specific location.

There is no doubt that these additional postulates describe what we experience when a measurement is made. But, now is the time for the promised mockery. While experimentally accurate, these postulates are not what one might expect from a physical theory. They look more like bizarre diktats produced by some relentless bureaucratic engine that happily engages in assaulting reason for its own ends.

The diktat does not actually tell you what "measurement" is - after all, what is it that we actually do when we make a measurement? All that is happening is that we take some quantum system made of atoms and we bring it in contact with a measuring device that is also made of atoms. Now we know that atoms interact with other atoms via the Schrodinger equation. But somehow when these atoms become part of a "measuring device", they suddenly decide to stop obeying the deterministic Schrodinger equation and decide to engage in random activities such as quantum state "collapse". After this temporary violation of

[1] To measure "reasonableness", one may count the number of unauthorized copies of the book in circulation.
[2] Without the aid of certain "medicines" that can now be legally procured in certain US states.

the Schrodinger equation, the atoms recover their good moral sense and go back into obeying it. The process of measurement is thus more like a trip that the atoms make to the great state of Nevada where certain laws of the United States are temporarily believed to be suspended.

How does an atom even know that it is part of a measuring device and can thus decide to ignore the Schrodinger equation? For example, if the hydrogen atom in a water molecule decided to "measure" the position of the electron in the adjoining oxygen atom, the water molecule would go berserk - the collapsed electron's position would imply that the water molecule is now in an excited state and no longer be the nice staid molecule that we love. But if the same Hydrogen atom was in a "detector" it is supposed to have the mysterious property of "collapsing" the quantum state of certain electrons that it has decided to measure (but of course, not the electrons in the detector itself). The sentient behavior of the Hydrogen atom implied by this postulate in fact lead to some unfortunate philosophical speculations by the early pioneers of quantum mechanics relating the humble field of physics to lofty pursuits such as the philosophy of consciousness. In more recent years, entrepreneurs have combined these poorly articulated concepts into profitable businesses, beautifully illustrating the fundamental reason why capitalism has proven to be such a remarkable engine for economic growth.

Suppose our atoms somehow do figure out that they are in Nevada and can temporarily stop obeying the Schrodinger equation. Now a part of the quantum state during the measurement ends up on the wrong side of the outcome and therefore has to suffer a shameful "collapse". How exactly is this erasure of their existence supposed to happen?

And finally, let us talk about macroscopic superpositions. We experimentally know that we can place atoms and molecules in superposition. We also know that a superconductor and a superfluid are quantum systems where electrons and nuclei are in macroscopic spatial superpositions. What is it that differentiates these superpositions from say a supposedly disallowed superposition such as a human being ending up in two places at the same time?

I think it is clear that the standard measurement postulates that we find in textbooks leave much to be desired. That is a polite euphemism and I have never been accused of politeness. So let me be blunt. It is pure nonsense.

3 Measurement from Schrodinger

I have just rudely proclaimed that the "measurement" postulates of quantum mechanics are "pure nonsense". Yet, they pretty obviously describe what an experimentalist sees when a measurement is performed. How am I going to explain that? The purpose of this section is the following: I am going to show that using only the deterministic Schrodinger equation and making one other assumption, I am going to explain all of the phenomenology of measurement. To do this, I am going to force you to accept that you (or, indeed, all objects) are made of atoms and thus subject to the rules of quantum mechanics. The main assumption I am going to make is that the Hamiltonians that we have in the world are local. What do we mean by a local Hamiltonian, or indeed a local operator? This terminology emerges from quantum field theory and it is tied to causality. In quantum field theory, the operators in the Hamiltonian are field operators and these are functions of position. So for example, if you had a scalar field theory, the Hamiltonian contains terms of the form $\int d^3x \phi\left(x\right)^n$ where $\phi\left(x\right)$ is the field operator of the scalar field. The statement that the Hamiltonian is local is the statement that in the operator $\phi\left(x\right)^n$, all the operators are evaluated at the same point x. We do not have operators of the form $\phi\left(x_1\right)\phi\left(x_2\right)$ with $x_1 \neq x_2$ in the Hamiltonian. What this means is that different physical systems can interact only if their wave-functions have spatial overlap and further that the interactions occur at definite points in space. It is not possible for a state at the point x_1 to influence a state at the point x_2 unless something physically communicates between x_1 and x_2. With this assumption, I can show how the phenomenology of measurement flows out of the Schrodinger equation. Let me list out the phenomenology that I am planning on explaining - this way you can check if I am being honest and indeed, if you think I am missing something, you can point it out:

1. How does measurement result in the phenomenology of the "collapse" of the wave-function?
2. Why does measurement seem to produce probability even though the Schrodinger equation is deterministic?

3. If the Schrodinger equation is all there is, I should be allowed to have macroscopic superpositions. Why am I thus not able to "see" such macroscopic superpositions?

4. There is no Born rule in the Schrodinger equation - where does that come from?

To show all this, I will pretend to be a fancy math type and prove a couple of lemmas. A properly trained lemma prover starts with some axioms and I would like to pretend that I am one. I will thus assume the one very important axiom that I have already confessed that I need - namely, I will assume that the Hamiltonians that we have in the world are *local*. This is an extremely important axiom. You will see that the correct way to think about measurement is that measurement is simply an interaction between different physical systems. The kinds of interactions that exist in the world therefore limit what kinds of outcomes one can get from a measurement. So when you think about measurements on your own and come up with some question like "Why can I not *see weird phenomenon X* allowed by linear quantum mechanics?", you should answer that question by asking, "Ok, if I want to see weird phenomenon X, what kind of Hamiltonian would I need? Is this Hamiltonian possible, given locality?".

The first lemma I will prove concerns the ease of orthogonality of the states of macroscopic bodies. Suppose we have a macroscopic system composed of N individual particles. In a world where Hamiltonians are local, there are just nearest neighbor interactions between these individual particles. This implies there isnt long range entanglement in the system[3]. In the absence of long range entanglement, the macroscopic quantum state $|\Psi\rangle$ of the system is simply a product state of the form $|\Psi\rangle = \Pi_i |\Psi_i\rangle$ where $|\Psi_i\rangle$ is the quantum state of the i^{th} particle. Let us now look at quantum states $|\Psi\rangle = \Pi_i |\Psi_i\rangle$ and $|\Phi\rangle = \Pi_i |\Phi_i\rangle$ of this system. Consider transition matrix elements of the form $\langle\Phi|H_I|\Psi\rangle$ where H_I is some local operator. Since the operator H_I is local, it can be expressed as $H_I = \sum_i O_i$ where O_i is the operator acting on the i^{th} particle and it is the identity on the rest of the system. The inner product $\langle\Phi|H_I|\Psi\rangle$ is equal to $\sum_i \left(\Pi_{i\neq j}\langle\Phi_j|\Psi_j\rangle\right)\langle\Phi_i|O_i|\Psi_i\rangle$. Now suppose I even gave you two states where for all the particles, we had nice big overlaps - that is $\langle\Phi_j|\Psi_j\rangle \approxeq 1$ for all j. Even so, the sum $\sum_i \left(\Pi_{i\neq j}\langle\Phi_j|\Psi_j\rangle\right)\langle\Phi_i|O_i|\Psi_i\rangle$ is approximately of the form $\sum_i \left(\langle\Phi_j|\Psi_j\rangle\right)^{N-1}\langle\Phi_i|O_i|\Psi_i\rangle$. But, for large N, $\left(\langle\Phi_j|\Psi_j\rangle\right)^{N-1} \approxeq 0$. Moreover, if some of the states in $|\Psi\rangle = \Pi_i|\Psi_i\rangle$ and $|\Phi\rangle = \Pi_i|\Phi_i\rangle$ actually happened to be orthogonal to each other, that is, if for some j, $\langle\Phi_j|\Psi_j\rangle = 0$, then for all but a small number of states, the terms $\left(\Pi_{i\neq j}\langle\Phi_j|\Psi_j\rangle\right)\langle\Phi_i|O_i|\Psi_i\rangle$ would vanish, vastly suppressing the inner product. Notice how easy it is for this latter condition to be satisfied. If there was a macroscopic system and it was interacting with a gas molecule, all you need is for the gas molecule to scatter off this macroscopic body. Upon scattering, the macroscopic body becomes entangled with different scattering directions of the scattered gas, which are all orthogonal to each other. As a consequence, this inner product will vanish! Thus, we have proven our first lemma, namely, the transition matrix elements $\langle\Phi|H_I|\Psi\rangle$ of the vast majority of quantum states $|\Phi\rangle$ and $|\Psi\rangle$ of a macroscopic system when evaluated on a local operator H_I is basically zero. Let us call this the Lemma of the Vanishing Inner Product, or the VIP[4] Lemma. This lemma will be important in describing the phenomenology of the "collapse" of the wave-function.

The second lemma that I will now prove concerns the nature of the quantum states that arise in a world where some specific quantum system interacts with a large number of particles. For example, suppose we had a large number of spins. We know that each spin sources a magnetic field. The collective magnetic field sourced by all these spins is some quantum state of the electromagnetic field. What kind of quantum state is it? The answer to this question are states called "Coherent States". You might have encountered them in your days as a quantum infant when the textbooks make you solve for states of "minimum uncertainty" or "states of well defined phase" for a quantum harmonic oscillator. Textbooks usually treat these states as curious parts of the Hilbert space of a harmonic oscillator. However, they are highly relevant to our macroscopic world. Why is a weird state of the Harmonic oscillator relevant for the macroscopic world? This is because quantum field theory describes the world - the usual single particle quantum mechanics that we learn is a limit of quantum field theory. Field theory is naturally constructed from a basis of harmonic

[3]In strongly coupled systems, it is possible to obtain long range entanglement, especially for the ground states of such systems. Those systems do exhibit unusual quantum properties. But, for even such systems, their typical state at normal temperatures does not exhibit long range entanglement. My interest in this note is to discuss how our classical experience of the world arises from the typical quantum states we find in our world - as opposed to the highly quantum behavior of particular states.

[4]Very Important Point

oscillator states and thus the coherent states of the harmonic oscillator can also describe important states of quantum fields. Now these coherent states have an extremely important property - they are completely described by the expectation values $\langle \Psi_c | \hat{A} | \Psi_c \rangle$ and $\langle \Psi_c | \hat{\Pi_A} | \Psi_c \rangle$ where $\hat{A}$ and $\hat{\Pi_A}$ are the field and its conjugate momentum operators respectively. The fact that the quantum states of the systems of interest (such as the electromagnetic field) will be coherent states when they are sourced by a number of interactions is going to be relevant to understanding the origin of the Born rule - the connection is going to come from the fact that these coherent states are specified simply by their expectation values on the appropriate operators. Let us now see why our quantum systems end up in these coherent states.

To figure out what quantum state a physical system will end up in, all we need to do is to be good obedient kids and solve the Schrodinger equation. So given a set of spins, if we want to figure out the state of the electromagnetic field produced by these spins, all we need to do is to put in the Hamiltonian of QED which contains the term $\int d^3x \bar{\Psi} \gamma^\mu \Psi A_\mu$ and take the incoming quantum state to be a state of a large number of spins and solve the Schrodinger equation. One can do this using standard Feynman diagram tricks, where one adds contributions coming from a number of spins. As a virtue of soft-photon theorems, one can then show that the final state produced by these spins is a coherent state of the electromagnetic field [1].

Incidentally, coherent states are not limited solely to electromagnetism. Suppose we had a macroscopic harmonic oscillator that was getting rung up due to external interactions - for example, we could think of a LC resonator in which a current gets generated due to the passage of electrons or spins next to the resonator. The leading order coupling between the charges in the LC resonator and the external interactions is some linear drive of the form $\hat{q} F(t)$ where $F(t)$ is the external driving term (coming from the transit of electrons or spins) and $\hat{q}$ is the operator controlling the charge degree of freedom in the resonator. One can show that when this oscillator is rung up, the state of the oscillator produced by this driving is also a coherent state of the LC resonator.

Ultimately, the math that proves the above results is applicable to any bosonic degree of freedom that is being excited by leading order interactions. I will call this lemma "Coherent State Dominance" to highlight the generic nature of coherent states in describing quantum systems that interact with a large number of particles.

3.1 The Phenomenology of Measurement

In this section, I want to describe how I, a fully biological organism, will experience the outcome of a measurement process. To be concrete, let us simply discuss the phenomenology of measurement in the context of the Stern Gerlach apparatus wherein I send various spins in the basis $|U\rangle$ and $|D\rangle$ (spins up and down along the z axis) and see what happens when I "measure" them. Since I need to describe what I "see", I have to provide a physical description of how I am physically able to see and react to what I see. Now this sounds like we need biology - but my knowledge of biology is rather poor[5]. So instead of real biology, we will do physicist's biology - that is, we will make up a toy model which should morally describe the way things actually work[6].

So here is the toy model. My brain is made of a large number of registers whose purpose is to record the outcome of spin measurements that I "see". For a particular register, before that register has seen any spin, it is in an initial quantum state $|0\rangle$. So if my brain was to only have the patience to see three spins, the initial state of my brain, before I have seen any spins will be $|000\rangle$. Next, when I see the state $|U\rangle$, the register $|0\rangle$ evolves to $|1\rangle$, and when I see the state $|D\rangle$, the register $|0\rangle$ evolves to $|2\rangle$. So for example, if I saw the first spin to be $|U\rangle$, the second also to be $|U\rangle$ and the third to be $|D\rangle$, the registers in my brain would be $|112\rangle$. Or if I have so far only seen the first spin and it is $|D\rangle$ and I haven't seen the other spins, the registers would be in the state $|200\rangle$. Now mind you, the rest of my body interacts with these spin registers. So if my spin register is $|112\rangle$ and the Hamiltonian of my brain was such that when the spin registers go from $|000\rangle$ to $|112\rangle$, I will go and play cricket, that is exactly what my biological being will do. You are about to complain that while I have told you what the spin registers are doing, I haven't yet described what I mean

[5]I was forced to take a Biology class at my undergraduate institution. I put this off till the very end of my time there and I eventually took it during the final quarter of my senior year. Due to infection by an advanced case of senioritis and the desire to explore Southern California, I ignored this class and graduated with a Gentleman's C+, the bare minimum necessary to graduate. I am forever thankful to my instructor Prof. Henry Lester for this gesture of kindness.

[6]A genuine physicist would disbelieve any claims from biologists that are contrary to the results obtained from this toy model.

by "seeing". Here is the toy model for "seeing" - this is simply an interaction Hamiltonian H_S between the registers of your brain and the spins which accomplishes the following time evolution:

$$e^{-iH_sT}|U\rangle|0\rangle \to |U\rangle|1\rangle \tag{1}$$

$$e^{-iH_sT}|D\rangle|0\rangle \to |D\rangle|2\rangle \tag{2}$$

Given this Hamiltonian, let us see how the brain responds if it was given a number of spins all of them in the state $|U\rangle$. The answer is pretty simple. We will get:

$$e^{-iH_sT}|UU\ldots U\rangle|00\ldots 0\rangle \to |UU\ldots U\rangle|11\ldots 1\rangle \tag{3}$$

Now, in addition to these spin registers, I am also going to assume that there is a predictivity circuit C_k that functions in your brain - the job of this predictivity circuit C_k is to interact with the first $k-1$ spin registers of your brain and see if it can guess what the next spin will be. If the guess is right, the predictivity circuit outputs a signal $|NP\rangle$ saying the world is not probabilistic. If the guess is wrong, the circuit outputs a signal $|P\rangle$ saying the world is probabilistic. Let us apply this circuit C_k on the state of the brain we have obtained when we sent in a bunch of $|U\rangle$ states to the brain. The answer is simple - we get:

$$C_k|UU\ldots U\rangle|11\ldots 1\rangle \to |UU\ldots U\rangle|11\ldots 1\rangle|NP\rangle \tag{4}$$

Thus, in this state, we see that there is no probability in quantum mechanics - we are getting completely deterministic results. In the parlance of the Official Postulates of Quantum Mechanics, this is the statement that the state $|U\rangle$ is an eigenstate of the operator being measured and thus one gets definite results.

That was the easy part where there is no wavefunction "collapse". Let us now see how we get that bizarre phenomenon from this simple toy model.

3.1.1 Wave-function Collapse:

To see wave-function "collapse", let us subject my brain to the spin state $\alpha|U\rangle + \beta|D\rangle$. The interaction Hamiltonian between the spins and the registers is still H_S. Since time evolution is linear, quantum mechanics robustly and easily predicts the answer. We have:

$$e^{-iH_sT}|\left(\alpha|U\rangle + \beta|D\rangle\right)|0\rangle \to \alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle \tag{5}$$

Notice that the final quantum state is **NOT** $|U\rangle|1\rangle$ **OR** $|D\rangle|2\rangle$. That would not be linear quantum mechanics. Rather it is the superposition of the two. What does this state mean? We initially started with a nice product state $(\alpha|U\rangle + \beta|D\rangle)|0\rangle$ which had a sensible interpretation - there is a spin state $(\alpha|U\rangle + \beta|D\rangle)$ and a separate state $|0\rangle$ which represents the quantum state of my brain. With a product state, the spin and my brain are factorized and one can ask questions such as "what is the state of my brain'", independent of the spin. But, due to linearity of quantum mechanics, the interaction has resulted in producing an entangled state $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$. In the entangled state, one cannot ask the question "what is the state of my brain" independent of the spin. The whole point of entanglement is that the quantum state of my brain is now correlated with the spin. So what we have is a bizarre state where the quantum state of my brain is entangled with the quantum states $|U\rangle$ and $|D\rangle$ of the spin.

Let us look at the time evolution of the quantum state $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$. Now in general, since this quantum state is in a superposition, one would expect interference between these states. It would thus seem that one cannot solely focus on the time evolution of just the state $|U\rangle|1\rangle$ or the state $|D\rangle|2\rangle$. But, for interference terms to matter, the Hamiltonian of the system must be such that the transition matrix elements $\langle U|\langle 1|H|2\rangle|D\rangle$ are non-zero. But, since H is a local Hamiltonian and I have a macroscopic brain[7], by the VIP lemma, this inner product is zero. This means that if we want to look at the time evolution of the quantum state $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$, we can simply time evolve the two states independently of each other.

This implies that the time evolution of $\alpha|U\rangle|1\rangle+\beta|D\rangle|2\rangle$ is equivalent to the time evolution of two distinct quantum states $|U\rangle|1\rangle$ and $|D\rangle|2\rangle$, even though the full quantum state is in fact the superposition of the

[7] There is much pleasure in writing papers when one gets to talk about the size of one's brain.

two. The quantum state $|U\rangle|1\rangle$ is a factorized state - it is the state where the spin is up and my brain's spin register has "seen" that the spin is up. The full quantum state also contains the state $|D\rangle|2\rangle$ where the spin is down and my brain's spin register has "seen" that the spin is down. Since these two states evolve independently of each other, if the Hamiltonian of my brain was such that the state $|U\rangle|1\rangle$ would time evolve into the rest of my body going to play cricket, than that is how that state will evolve. And if the state $|D\rangle|2\rangle$ meant that I would go get Indian food for dinner[8], then that is what will happen. So what we have is:

$$e^{-iH_sT}|\left(\alpha|U\rangle+\beta|D\rangle\right)|0\rangle \rightarrow \alpha|U\rangle|1\rangle+\beta|D\rangle|2\rangle \rightarrow \alpha|U\rangle|\text{SR plays cricket}\rangle+\beta|D\rangle|\text{SR eats Indian Food}\rangle \quad (6)$$

with these states subsequently evolving independently of each other. Now since the state $|U\rangle|1\rangle$ can ignore the existence of $|D\rangle|2\rangle$ in its subsequent evolution, this state will simply say that the wave-function "collapsed" to $|U\rangle|1\rangle$. But of course, this is simply a highly effective description of the system which holds in the limit $\langle U|\langle 1|H|2\rangle|D\rangle=0$. If by some hook or crook, this inner product becomes non-zero, the "collapse" would no longer be true and the two quantum states will influence each other's subsequent behavior. This part of the description of measurement where there is effective wave-function collapse is typically called "decoherence". So when you hear people say "doesn't decoherence describe measurement", this is what they mean and they are correct.

3.1.2 Probability from Determinism:

Let me now show how the deterministic Schrodinger equation yields the perception of probability. Notice that the time evolution that produced (5) is deterministic - there is no doubt that this is the final state produced by the Schrodinger equation. To see probability, let us observe a second spin that is also in the quantum state $\alpha|U\rangle+\beta|D\rangle$. We will again use the same Hamiltonian H_s to do the time evolution of the system. The Schrodinger equation then predicts:

$$e^{-iH_sT}|\left(\alpha|U\rangle+\beta|D\rangle\right)\left(\alpha|U\rangle|10\rangle+\beta|D\rangle|20\rangle\right) \rightarrow \alpha^2|UU\rangle|11\rangle+\alpha\beta|UD\rangle|12\rangle+\beta\alpha|DU\rangle|21\rangle+\beta^2|DD\rangle|22\rangle \quad (7)$$

Here I have kept track of the fact that I have already interacted with the first spin and thus the spin registers in my brain for that interaction have already evolved into their respective states. The spin register for the second spin is initially in the state $|0\rangle$ and that gets updated as a result of this interaction. Once again, this is a deterministic result. But, due to decoherence, we know that the states $|UU\rangle|11\rangle$, $|UD\rangle|12\rangle$ *etc.* evolve independently of each other. And of course, each of these states comes equipped with the rest of the machinery of my brain, including the predictivity circuit C_k. Suppose the quantum state $|UD\rangle|12\rangle$ invokes its predictivity circuit C_2. This circuit is supposed to look at the state of the first spin register and see if it can guess the next one. But, for the state $|12\rangle$, knowing that the first spin register is in state $|1\rangle$ does not tell you the state of the second spin register. If I now interact with a large number of spins, the quantum states of my brain will be various sequences like $|121122\ldots\rangle$ where if that state runs the predictivity circuit C_k on it, knowing the values of the first $k-1$ registers is no guarantee that the circuit can predict the value of the k^{th} register. The predictivity circuit will thus say that the outcomes of spin measurements is probabilistic. We thus see how the experience of probability has emerged from the deterministic Schrodinger equation. The deterministic time evolution of the system produced a well defined final state and in this final state all the outcomes of the interaction occurred. However, as a result of decoherence, the different states (outcomes of the interaction) do not influence each other and thus each state evolves effectively with a probabilistic description of the outcome of measurement.

3.1.3 Why Can't we "see" Macroscopic Superpositions?

Given that linear quantum mechanics robustly predicts the existence of superpositions and the fact that quantum mechanics claims it can be applied to systems of any size, why are we not able to "see" macroscopic superpositions? In other words, everybody in this planet believes that there is a state of the universe where Surjeet Rajendran is sitting on his LazBoy in his house. This nice quantum state is a solution to the

[8] More likely than me playing cricket these days

Schrodinger equation. There is also another solution to the Schrodinger equation where Surjeet Rajendran is sitting on his LazBoy in his office. The linearity of the Schrodinger equation implies that the superposition of these two states of Surjeet Rajendran is also a solution to the Schrodinger equation and is thus an allowed state of quantum mechanics. If this is the case and if we are able to "see" this state of Surjeet Rajendran, then all of these meandering arguments about quantum mechanics and its interpretations would be over. We would just know the answer. Why is this difficult, or in fact, nearly impossible within linear quantum mechanics?

To answer this question, let us look at the example of the spin where there is no doubt that there are superpositions. For example, when we are given the spin state $\alpha|U\rangle + \beta|D\rangle$, if we interacted with it using the Hamiltonian H_s like we had been doing, the time evolution always gives the complicated entangled state $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$. In this state, there is always probability involved in the effective description of the system and in fact, as we have seen, after the interaction, due to decoherence, the spin states evolve as though the wave-function has "collapsed". The states of my brain are only able to see the states $|U\rangle$ or $|D\rangle$ but not the superposition $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$. But, for spins, this issue is easily rectified. The reason why we are only able to see the states $|U\rangle$ or $|D\rangle$ is because our interaction Hamiltonian H_s was of the form (1) and (2). A reasonable form of H_s that would give such a time evolution is a magnetic field oriented along the z direction, a pixellated screen which lights up when the spin hits the screen, photon receptors in my eye that get triggered when light from the screen hits my eye and my spin register changing as a result of this trigger. Now if I want to "see" the state $\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$ what I want is a new interaction Hamiltonian H_n of the form:

$$e^{-iH_nT}\left(\alpha|U\rangle + \beta|D\rangle\right)|0\rangle \rightarrow \left(\alpha|U\rangle + \beta|D\rangle\right)|1\rangle \tag{8}$$

$$e^{-iH_nT}\left(-\beta^*|U\rangle + \alpha^*|D\rangle\right)|0\rangle \rightarrow \left(-\beta^*|U\rangle + \alpha^*|D\rangle\right)|2\rangle \tag{9}$$

This interaction Hamiltonian H_n is easy to create - all we need to do is to orient our magnetic field along the direction (α, β) in the Bloch sphere and then use the rest of the apparatus (screen, eyes and brain) as is. The reason why this is easy to do is precisely because the spin is a local object and there are local interactions between the spin and a magnetic field allowing us to create H_n. With this understanding, let us now answer the question about "seeing" Surjeet Rajendran. Let us consider two states of Surjeet Rajendran $|SR_H\rangle$ and $|SR_O\rangle$ corresponding to me being in my home and office respectively. Why is everyone able to see me in my home and in my office? That is because there is an interaction Hamiltonian H_{SR} between the atoms in my body and your brain (I will abuse notation and use the states $|0\rangle$, $|1\rangle$, $|2\rangle$ to describe states of your brain as well) which permits the time evolution:

$$e^{-iH_{SR}T}|SR_H\rangle|0\rangle \rightarrow |SR_H\rangle|1\rangle \tag{10}$$

$$e^{-iH_{SR}T}|SR_O\rangle|0\rangle \rightarrow |SR_O\rangle|2\rangle \tag{11}$$

Such an interaction Hamiltonian clearly exists because the atoms in my body are able to emit (or reflect) light and this light can go into the photon receptors of your eyes. These are all local operators. Now to "see" Surjeet Rajendran in a superposition of being in my home and in my office, what you need is a new interaction Hamiltonian H_{SP} which permits the time evolution:

$$e^{-iH_{SP}T}\left(\alpha|SR_H\rangle + \beta|SR_O\rangle\right)|0\rangle \rightarrow \left(\alpha|SR_H\rangle + \beta|SR_O\rangle\right)|1\rangle \tag{12}$$

$$e^{-iH_{SP}T}\left(-\beta^*|SR_H\rangle + \alpha^*|SR_O\rangle\right)|0\rangle \rightarrow \left(-\beta^*|SR_H\rangle + \alpha^*|SR_O\rangle\right)|2\rangle \tag{13}$$

Does such a Hamiltonian H_{SP} exist? If my axioms only involved linear quantum mechanics, one can indeed construct such an operator. But the key point is that in addition to linear quantum mechanics, I am restricting the kinds of operators that exist in the world to **local** operators. That is, physical systems need overlap at the same spatial point in order to interact - without such overlap, there is no interaction. The point about the operator H_{SP} above is that it needs to simultaneously know about the quantum states $|SR_O\rangle$ and $|SR_H\rangle$ which are two quantum states localized at two very different locations - any H_{SP} that is able to act on both these states simultaneously is a non-local operator. Our requirement of locality forbids the existence of such an operator and thus we do not have interaction Hamiltonians of the form H_{SP} that would allow us to see Surjeet Rajendran at two different locations at the same time.

3.1.4 The Emergence of the Born Rule:

Finally, let us now see how the Born rule emerges from this description. The Born rule is the statement that when the quantum state is of the form $\alpha|U\rangle + \beta|D\rangle$, then when we measure the spins in the basis $|U\rangle$ and $|D\rangle$, the probability that the spin appears as $|U\rangle$ is $|\alpha|^2$ and the probability that it appears as $|D\rangle$ is $|\beta|^2$. Now probability is of course a quantity that makes sense only in the limit of a large number of spins being measured. That is, we should measure N spins and count the number of times we got $|U\rangle$. In the limit of large N, the ratio of the number of measurements with $|U\rangle$ over the total number of measurements approaches $|\alpha|^2$.

It will turn out that the above definition of the Born rule is not quite true in quantum mechanics. We will find that the Born rule emerges when the measurement apparatus has some non-zero resolution so that it is not sensitive enough to resolve the outcome of individual spins but is rather sensitive to the outcomes of a collection of them. Of course, nothing stops us from coming up with a measurement apparatus which is in fact sensitive to individual spins - in this case we will find that there will be outcomes produced by the measurement that do not experience the Born rule. But, in the limit of large N, the set of such quantum states where the Born rule is violated will be a set of measure zero.

To see how this works, let us begin with a simpler measurement protocol where we do not have sensitivity to resolve the outcomes of individual spins, but can see their collective coarse-grained effects. Let us take N spins, each in the state $\alpha|U\rangle + \beta|D\rangle$. We allow the states to go through a magnetic field oriented in the z direction because of which the spins separate in physical space, with the states $|U\rangle$ ending up at the spatial point $|T\rangle$ and the states $|D\rangle$ ending up at the spatial point $|B\rangle$. We send all the N spins through this magnetic field and obtain a final state. We then put a SQUID magnetometer around this entire system and we look at the current in the SQUID, thus measuring the total magnetic field of the spin states. What magnetic field will we see?

Now each spin term evolves as:

$$\alpha|U\rangle + \beta|D\rangle \rightarrow \alpha|U\rangle|T\rangle + \beta|D\rangle|B\rangle \tag{14}$$

When I send in N spins, the final state is of the form:

$$(\alpha|U\rangle + \beta|D\rangle)^N \rightarrow \alpha^N|UU\ldots U\rangle|TT\ldots T\rangle + \alpha^{N-1}\beta|U\ldots UD\rangle|T\ldots TB\rangle + \ldots \tag{15}$$

In this expression, there are $\binom{N}{k}$ terms that have the coefficient $\alpha^k\beta^{N-k}$ with k $|U\rangle$ spins at positions $|T\rangle$ and $N-k$ $|D\rangle$ spins at positions $|B\rangle$. Let us write this quantum state in the symbolic form:

$$|\Psi\rangle \approxeq \sum_k \binom{N}{k}\alpha^k\beta^{N-k}|\underbrace{U\ldots U}_{k}\rangle|\underbrace{D\ldots D}_{N-k}\rangle|\underbrace{T\ldots T}_{k}\rangle|\underbrace{B\ldots B}_{N-k}\rangle \tag{16}$$

Even though I wrote this in the above manner, the $\approxeq$ sign symbolizes the fact that this is not the strict superposition - I have clubbed in all the states that have k $|U\rangle$ together as though they were all the same state - but this is not true. The order does matter and this way of writing the state is simply a way of gathering all the spins together. Now you might be tempted to look at the above result and see hints of the binomial probability distribution show up there. But this is wrong - first, this would give the probability as α and β - which we know is not correct. The reason why the binomial probability distribution does not arise here is because the α and β are complex, instead of being non negative real numbers.

Now given this final state for the spins, what we are doing is exposing these spins to the SQUID. The Hamiltonian of the SQUID is such that the current in the SQUID is coupled to the magnetic field produced by these spins. In the limit of large N, as a result of Coherent State Dominance, the state of the electromagnetic field sourced by these spins is a coherent state. This coherent state is completely characterized by the expectation value $\langle\Psi|\vec{A}|\Psi\rangle$. This expectation value is:

$$\langle\Psi|\vec{A}|\Psi\rangle = \sum_k \binom{N}{k}|\alpha|^{2k}|\beta|^{2(N-k)}\vec{A}\left((k,T),(N-k,B)\right) \tag{17}$$

where $\vec{A}\left((k,T),(N-k,B)\right)$ is the classical vector potential of a system where k spins are at the position T and $N-k$ are at the position B. This expectation value clearly is the binomial distribution with probability $|\alpha|^2$ for the spin to be at T and $|\beta|^2$ for it to be at B.

The SQUID will thus respond as though it was seeing the magnetic field of a classical probability distribution where the probability of getting spin $|U\rangle$ was $|\alpha|^2$ and the probability of seeing spin $|D\rangle$ was $|\beta|^2$. We thus see where the Born rule emerges - it comes from the fact that the state of the counting device we use, namely the state of the electromagnetic field in this example, is in a coherent state and that this coherent state is completely characterized by its expectation value.

Notice however that the SQUID simply interacts with the overall magnetic field produced by this collection of spins. This is thus a coarse-grained measurement of the full distribution of spins. The SQUID itself is not entangled with any particular state that is produced in the massive superposition (15). Let us now take the opposite limit. What if we had used a measurement apparatus where we were able to resolve each individual spin state produced in (17). For example, we put a screen at the locations T and B and when the spin hits T, the screen emits red light and when the spin hits B, the screen emits blue light. I then see this light and thus I am entangled with the specific outcome. In this case, the superposition we produce is of the form:

$$|\Psi\rangle = \alpha^N |U\dots U\rangle|1\dots 1\rangle + \alpha^{N-1}\beta|U\dots UD\rangle|1\dots 12\rangle + \cdots + \beta^N|D\dots D\rangle|2\dots 2\rangle \tag{18}$$

Now each of these states evolves independently of the others. How do we see probability in this specific outcome? In a specific outcome, the naive notion of probability, where we count the number of spins that are $|U\rangle$ and take the ratio, is a counterfactual question. That is, we get some answer for this ratio and in the limit of large N, we believe that we are not lucky to have obtained this ratio. In other words, if we look at the distribution of outcomes, then in most of this distribution, the ratio would be the true probability. But, in linear quantum mechanics, there is no way for us to "talk" to the rest of the distribution and verify this counterfactual claim.

If we are not able to "talk" to the rest of the distribution, what do we have instead? Notice that we now have something quite odd. The deterministic measurement process produces all the possible outcomes of the measurement. It thus produces extreme states of the form $|U\dots U\rangle|1\dots 1\rangle$ and $|D\dots D\rangle|2\dots 2\rangle$ in which my brain only sees either $|U\rangle$ or $|D\rangle$ even though the spin that I sent was of the form $\alpha|U\rangle + \beta|D\rangle$, in which case, I should have expected to see a distribution of spins. These states that are produced as a result of this measurement procedure will thus not experience the Born rule. There are of course plenty of states where the Born rule will in fact also be observed to be true. In this context, the appropriate question to ask is the measure of the set of states where the Born rule is violated. On the Hilbert space, the natural measure to use is the inner product measure - and this yields the same formula as the expectation value, wherein we see that these extreme states are weighed down by the coefficients $|\alpha|^{2N}$ and $|\beta|^{2N}$ respectively. In the limit of large N, one can again show that on the measure in Hilbert space, the set of all states that deviate away from the Born rule is zero [2].

Given these two very different possibilities, let us talk about what we in fact experimentally do when we are trying to measure probabilities in quantum experiments. The devices we use are rarely sensitive to the outcomes of each individual particle we are measuring. What we usually do is to put a device that aggregates a bunch of outcomes and yields a result that is coarse grained at that level. So for example, in the case of spins that hit a screen, even though single photon detectors do exist, the detectors we typically use require us to collect a bunch of photons before they respond. In this case, what we are effectively doing is taking our set of N spins, dividing them into blocks of size M each. Now in each block, we are performing a coarse grained measurement, and then we can compare different coarse grained measurements. The Born rule will reappear for this coarse grained measurement (it clearly does when we use a SQUID - see below for the argument for why it would also appear if we used a different device). One can then compare the outcomes of different coarse-grained measurements many times to see that the Born rule continues to hold. The important realization that we get from this discussion is that the experimental experience of the Born rule is tied to the fact that the measurements we perform typically involve an element of coarse graining - if we had truly fine grained measurements down to the single particle level, there will be quantum states, of vanishing measure, where the Born rule will never be realized.

This brings up the key point. Are we required to use a SQUID that couples to the electromagnetic field in order to see the Born rule? After all, that is not what we usually do[9]. But, no matter what, we need to have a physical counting apparatus that is able to go into some well defined state when it encounters a number of spins. This apparatus contains a degree of freedom X which is coupled to the spins. X needs to be a bosonic degree of freedom since it needs to be put into high occupation number states so that it can count a large number of spins. To leading order, X is a harmonic oscillator and its leading interactions with the spins is going to be via a linear driving term. Due to Coherent State Dominance, we will once again find that X will end up in a coherent state, at which point, it will be described by the expectation value $\langle\Psi|X|\Psi\rangle$, resulting in the emergence of the Born rule.

Now, in my arguments, I have made heavy use of electromagnetism and harmonic oscillators. Is the Born rule specific to these interactions, or is it more general? Now, the fact of the matter is, electromagnetism and harmonic oscillators pretty much describe every physical system we actually care about. So very reasonably, this covers all physical cases of interest. But, I am an academic and it is interesting to ask academic questions about what might happen for a different system. While it is difficult to prove useful results about generic systems that aren't sufficiently defined, one can ask what sorts of features a generic system needs to have so that it is a good counting device and see if those aspects also make it susceptible to the key aspect of Coherent State Dominance, namely, the fact that the response of the counting system is determined by the expectation value of an appropriate operator. Now, a good counting apparatus needs to be able to retain information about whatever it is counting even when it is subject to a bunch of environmental disturbances. An easy way for a system to resist the environment and maintain information is by being heavy (or containing a macroscopic number of particles). For example, our counting apparatus could be the position of some kind of heavy (compared to atomic scales, but still small compared to macroscopic masses) particle. In a case like this, the de-Broglie wave-length of the particle will be small. In this case, very reasonably, the spread of the particle's wave-function can be significantly smaller than the length scale over which the potential $V(x)$ of the particle changes. If we now consider the quantum mechanics of this system interacting with a bunch of other quantum particles, due to Ehrenfest's theorem, we know that the expectation values of various quantum operators automatically obey the corresponding classical equations of motion. When the de Broglie wavelength is smaller than the length scale of the variation of the potential, we can replace $\langle\Psi|V(\hat{x})|\Psi\rangle$ with $V(\langle\Psi|\hat{x}|\Psi\rangle)$ (similar replacements can also be performed for the derivatives of $V(x)$) and thus Ehrenfest's equations directly become equations for $\langle\Psi|\hat{x}|\Psi\rangle$ sourced by the expectation values of the spins. When the spread of the wave-function is small, the expectation value $\langle\Psi|\hat{x}|\Psi\rangle$ is a representative of the state of the system and this is the value that acts as the counting apparatus. This is of course the same kind of behavior that we saw for coherent states.

4 Conclusions

I hope I have convinced you that quantum mechanics is in fact a complete theory without need for ad-hoc measurement postulates that do not make any kind of logical sense. All one really needs is the Schrodinger equation, coupled with the additional assumption that the Hamiltonians that exist in the world are local. These two facts are sufficient to derive the phenomenology of measurement such as the "collapse" of the wave-function, the emergence of probability from a deterministic time evolution[10] and the fact that it is difficult to observe superpositions of macroscopic bodies. Further, once we recognize that the measuring apparatuses that we use to count the outcomes of measurement are in coherent states (or otherwise suitably localized), we can also see how the probabilities that emerge in this deterministic evolution obey Born's rule.

Notice how conservative this entire derivation is - I have only used the Schrodinger equation and I have insisted that all atoms, including the ones that you are made of, are subject to the rules of quantum mechanics and thus capable of being placed in entangled states. It is true that I had to make some informed toy models about biology - and one may not like the fact that biological elements seem necessary to describe physics. Indeed, the toy model of biology here is a mirage - you can replace that toy biological model with any kind of macroscopic object or detector. All we are really asking is how a quantum system can get entangled

[9] My brain is sometimes nourished by biological squids, especially the deep fried and salty kind. But, as far as I know, even though I have received multiple vaccinations in my lifetime, I do not believe that a SQUID was secretly implanted into my brain.

[10] Einstein was in fact right - God does not play dice with the universe. We are just too decohered to see God's master plan of deterministic evolution

with a macroscopic system. Once this entanglement has been created, the macroscopic system (whether sentient or not) will effectively evolve as though there was wave-function “collapse”. Further, given that this macroscopic system is going to be in some kind of coherent state in order to preserve its structure in the face of environmental decoherence, the macroscopic system will react to the expectation value of the quantum stimuli that it is subject to, exactly as described by the Born rule.

References

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