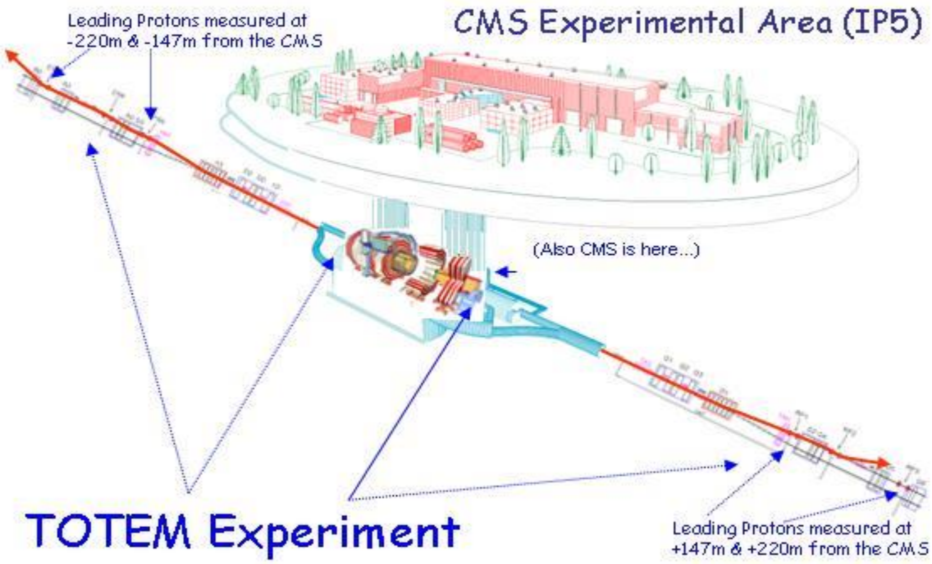
TOTEM 2: INTERFERENCE
TEACHER NOTES

DESCRIPTION

The *TOTal cross section, Elastic scattering and diffraction dissociation Measurement* (TOTEM) experiment is designed to understand the elastic collisions of protons in the Large Hadron Collider (LHC) at CERN. As stated on the TOTEM website, "TOTEM's physics program is dedicated to the precise measurement of the proton-proton interaction cross section, as well as to the in-depth study of the proton structure which is still poorly understood." TOTEM detectors are installed just adjacent to the LHC beamline far forward (220 m) on either side of the Compact Muon Solenoid (CMS) detector. While CMS looks at the results of elastic and inelastic scattering of quarks and gluons in near or completely head-on collisions of protons of which they are components, TOTEM looks at the results of more glancing collisions from which the protons emerge intact. The scattering of these protons is at extremely shallow angles, generally on the order of one ten-thousandth of a radian or a thousandth of a degree.



https://www.lhc-closer.es/taking_a_closer_look_at_lhc/0.totem

The results of this experiment give students and teachers the opportunity to investigate what happens when quantum objects like protons non-destructively interact. These investigations are presented in three parts which can each stand alone or can work together as a unified whole.

In this second part, students find the scattering angles of protons that collide at Point 5 at the Large Hadron Collider (LHC) using event displays. They then plot a histogram of the scattering angles to uncover evidence of the quantum wave nature of the proton. You should note that this wave nature is shown using a statistical aggregate of large data sets. Students can use this model along with the principles of wave interreference studied in classical physics to estimate an upper limit of the proton diameter.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

4. Analyzing and interpreting data

5. Using mathematics and analytical thinking
8. Obtaining, evaluating and communicating information

Common Core Literacy Standards

Reading

9-12.4 Determine the meaning of symbols, key terms . . .
9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

ENDURING UNDERSTANDING

- Fundamental particles display both wave and particle properties, and both must be taken into account to fully understand them.
- Scientists use models to make predictions about and explain natural phenomena.

LEARNING OBJECTIVES

Students will know and be able to:

1. Create and interpret a histogram from data.
2. Examine the histogram to observe wave interference characteristics.
3. Apply the de Broglie wavelength to estimate the size of a proton.
4. Discuss the limitations of models used in this activity.

PRIOR KNOWLEDGE

Students must be able to

- Plot and interpret a graph from data.
- Identify features of a graph indicative of wave interference.
- Make connections between momentum and energy in classical mechanics.

BACKGROUND MATERIAL

These resources on TOTEM, the LHC and the physics related to the activity are included on the student pages:

- LHC: The Large Hadron Collider video: https://www.youtube.com/watch?v=debQ60QVtYQ.
- TOTEM on their website: http://totem-experiment.web.cern.ch/totem-experiment/ and in the video: https://www.youtube.com/watch?v=YsZhwu32Zaw.
- de Broglie wavelength: http://en.wikipedia.org/wiki/Matter_wave
- Diffraction: http://en.wikipedia.org/wiki/Diffraction
- Proton radius: https://pdglive.lbl.gov/DataBlock.action?node=S016CR.

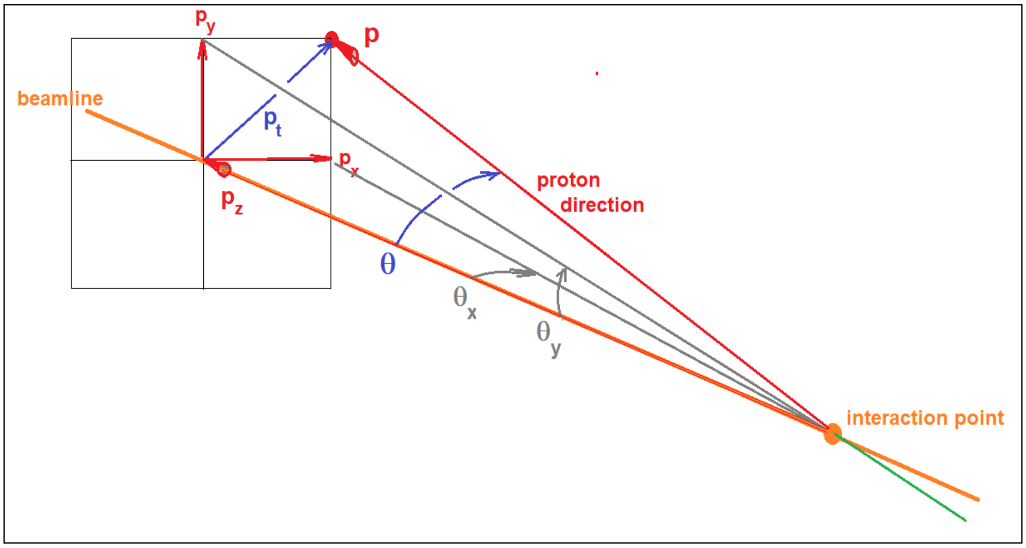
RESOURCES

Data file: https://quarknet.org/sites/default/files/totem_events_sm_1.pdf
Tally sheet: https://quarknet.org/sites/default/files/totem_tally_0.pdf

Provide a method for the whole class to make a histogram from -240 μrad to +240 μrad with a 10 μrad bin width (see tally sheet). A preferred approach is to create a space onto which students add a post-it for each event in the appropriate bin of a histogram with scattering angle on the horizontal axis and number of events on the vertical axis. See the next section for an example.

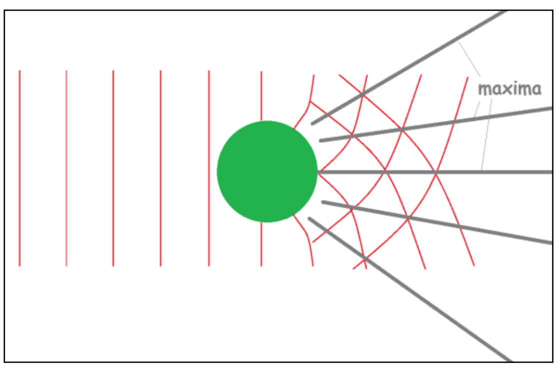
Physics and Geometry Discussion

The angles in the data events are directly related to the momentum vector. This relationship is explained in the figure below:



The momentum vector for a proton is shown as the red arrow from the interaction point, where the two protons collide, to the red dot on the blue double circle (not shown). The momentum vector makes an angle θ with the beamline. The components of momentum ***p****,* ***p****$_x$ and* ***p****$_y$, make angles θ_x and θ_y, respectively, with the beamline.*

The key data collection hinges on determining the angle between the proton direction and the beamline. We will use θ rather than θ_x and θ_y to describe our results. The method of finding θ from the event image is discussed in the Implementation section.



When the students make a histogram from the data, the histogram looks like an interference pattern with a gap in the middle. This gap occurs in the beam pipe region, where no detector components are present. You may have to guide your students to realize the significance of the gap. Taking the gap into account, the resulting interference pattern will be familiar to students who have studied double slit or single slit interference. This interference pattern provides evidence for the claim that a proton can act both as a wave and a particle. Each of the protons acts as a barrier (particle) for the other proton creating the interference pattern (wave) as shown in the diagram to the left. Therefore, the size of the proton can be estimated using the classical model for interference of a plane wave around a barrier.

The classical equation for such interference is

$$\sin\theta_n = n\lambda/d,$$

where n = 0, 1, 2, ... and represents the order of the maxima. Angle θ_n is the angle of each maximum. Note that for the central maximum n=0 and $\theta_0=0$. The variable d represents the diameter of one proton and λ represents the wavelength of the other proton.

You find wavelength λ of the proton using the beam momentum and Planck's constant using the deBroglie relation $\lambda = h/p$. You can calculate λ and use the result in the interference equation to solve for the proton diameter d. Your result for d will be within an order of magnitude or so of the published value of the proton diameter. This result provides evidence for the dual nature of matter. A sample calculation is shown in Implementation section below.

IMPLEMENTATION

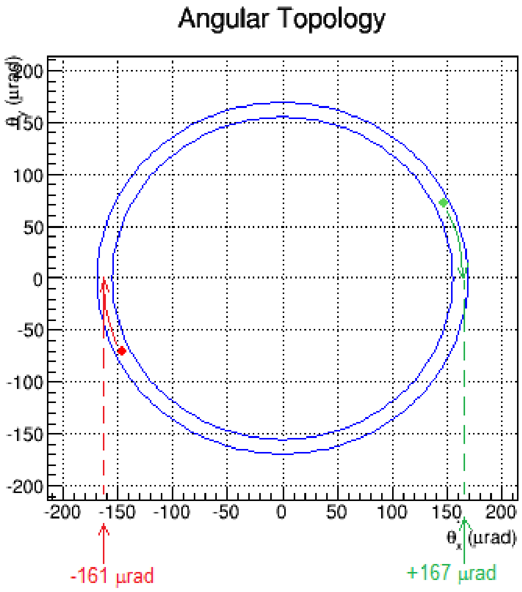
Part 1:

The beam for these data has an energy of 4 TeV which corresponds to a momentum of 4 TeV/c. With this information, your students can calculate the wavelength of 4 TeV protons. They can do this calculation as homework before the activity or at the beginning of the activity; the result should be around 3.2×10^{-13} μm.

The data file contains 21 pages of events, 2 events per page. Students work in pairs to analyze six or more events. The data file is in pdf format so you can print the events or students can view them on their devices. Plan to oversample the data—more than one pair of students analyzes the same data—as this will ensure greater precision of results.

The data you need to collect is the angle between the proton direction and the beamline. The event display shows the proton position in a pair of blue concentric circles. The value of this angle θ is the same for any point around the circle. The easiest place to determine the angle is when the circle intersects the x-axis. Therefore, your students need to rotate their data point to the $\theta_y = 0$ line and then drop a perpendicular line to read the θ_x value. The θ_x value at that point is the desired angle θ.

As shown below, each event has a red dot and a green dot. The red dot and green dot represent scattered protons detected by TOTEM 200 m from the interaction point, each traveling in opposite directions along the beam line. Find θ for each dot and record it in your data table.



The sample event to the left shows the main features of an event. The details of each event will vary.

The TOTEM detectors are embedded into the edges of the LHC beam pipe. Each records a "hit" where a scattered proton strikes. The red and green dots represent these hits.

The event display shows a planar view, with the beam occupying an area in the middle; there is a gap in the detector there. By tracing each dot around the circle to the "θ_x-axis," we can read the value of scattering angle θ in μrad. See the figure on the left.

In any given event, the values for θ for the red and green dots are very close to each other but opposite in sign. If you guide students to notice this, they might be able to correctly conclude that this is a result of conservation of momentum. This also means that if we measure the momentum of a proton at the detector on one side of the collision, we know the momentum of the proton on the other side, over 400 m away as discovered in the TOTEM 1 activity.

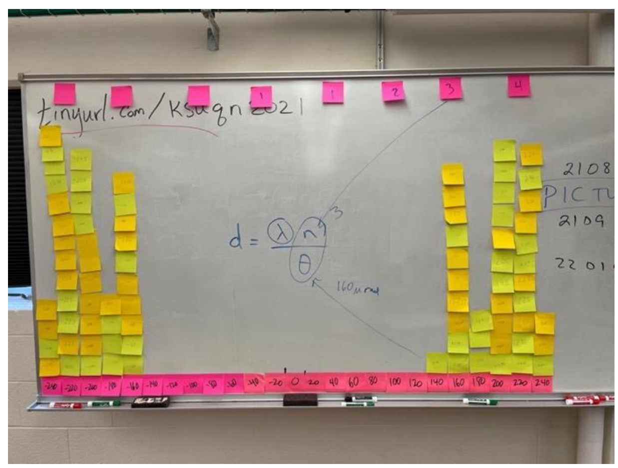
Instruct your students to record results in the tally sheet. See the example below. In this sample tally sheet students have examined six events. The students identified and recorded a positive and negative scattering angle θ for each event. Notice that each bin is 10 μrad wide centered on a multiple of 10; students round to the nearest multiple of ten to record the data. The negative θ values are for the red dots and positive θ values are for the green dots. Students do not need to color code results. You may guide your students to note that the middle of the distribution is empty because the detector cannot enter the region of the beam. Using the "blocked off angles and a distance of 220 m, students can estimate an upper limit to the width of the LHC beam. They use this gap and the angular distance between adjacent peaks to estimate the value of n for each peak; they use n, θ, and λ to calculate an upper limit to the diameter d of the proton. Students can do this in small groups, each using a peak, and then compare the results.

How wide is my proton? Data Tally Sheet

Make a mark for each event at its nearest value of θ. When finished, add the tally marks in each bin.

theta, μrad	Events	theta, μrad	Events
-240	I	10	
-230		20	
-220		30	
-210		40	
-200	I	50	
-190	I	60	
-180	I	70	
-170		80	
-160	II	90	
-150		100	
-140		110	
-130		120	
-120		130	
-110		140	
-100		150	
-90		160	I
-80		170	I
-70		180	I
-60		190	
-50		200	II
-40		210	
-30		220	
-20		230	
-10		240	I
0			

Students use their results, the number of data marks they have at each scattering angle, to fill in a histogram for the whole class. One way to do this is to use "sticky notes" on a prepared space, as below. Notice that in this histogram, the bin width is 20 μrad. You may select bin widths for the histogram of 10 μrad or 20 μrad.



Results from the 2021 Kansas State University QuarkNet Center.
Note the gap in the data representing a physical limit of the detector.

You can start the analysis by facilitating a discussion about patterns in these data and what the patterns may imply. If you have covered wave interference, the students may notice that the evenly spaced peaks are analogous to the maxima in an interference pattern.

Part 2:

Assumptions of the model:

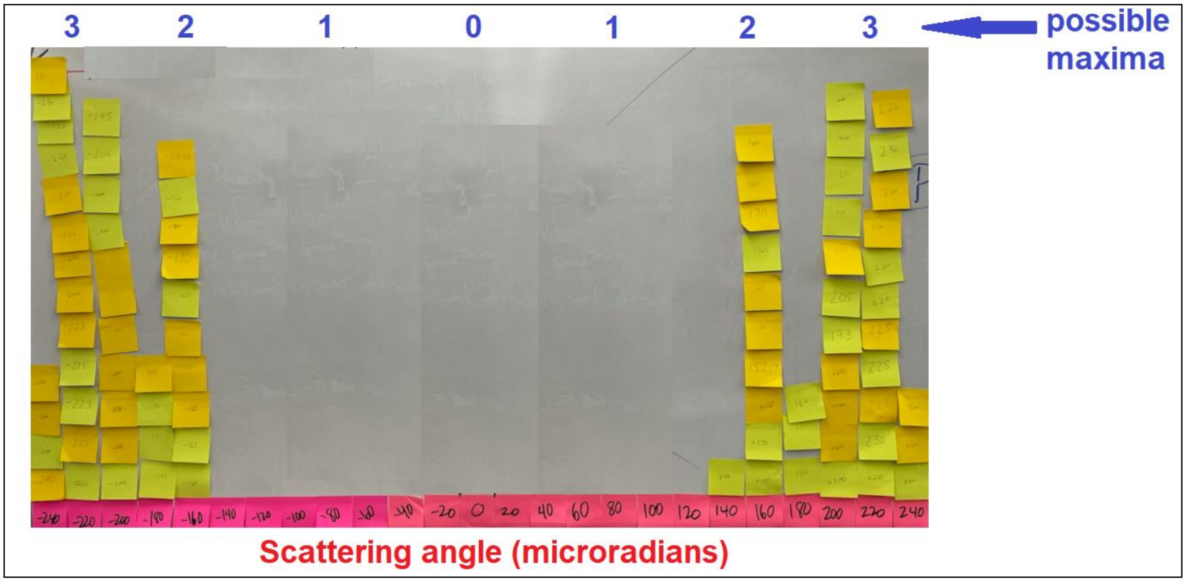
- The proton is a quantum object, so the radius of a proton is not like the radius of a marble.
- When protons collide in elastic collisions, one proton acts as a barrier around which the other proton can diffract.
- The proton, as a wave, is diffracted in two dimensions similar to a wave diffracting around a thin barrier.
- The barrier width represents the proton diameter.

This activity leads to a result that is correct within an order of magnitude (power of 10). Physicists often use existing equipment and data to project further results. It is not a calibration but a check on assumptions. Therefore, the students can be encouraged to have a discussion about how refining assumptions can lead to more precise results.

Physicists normally measure the diameter of a proton by scattering electrons to determine the "charge" radius, which identifies where the electrons are scattered by the proton. When physicists analysis of the data from TOTEM includes quantum effects such as the wave nature of matter to derive their result. The analysis in this activity uses a classical wave interference solution rather than more difficult quantum calculations. A result that is within an order of magnitude of the charge radius is to be expected. In our experience, students will determine a value for the diameter of the proton approximately 2-5 times the published proton diameter of about 1.8 fm or 1.8×10^{-15} m.

SAMPLE CALCULATIONS

Determine the wavelength of a 4 TeV proton using the deBroglie relation $p = h/\lambda$, or $\lambda = h/p$.
Planck's constant $h = 4.1 \times 10^{-15}$ eV•s (equivalent to the MKS value of 6.6×10^{-34} J-s).
The proton momentum, 4 TeV/c = $(4 \times 10^{12}$ eV$)/(3 \times 10^{8}$ m/s$) = 1.3 \times 10^{4}$ eV•s/m.
Thus $\lambda = h/p = (4.1 \times 10^{-15}$ eV•s$)/(1.3 \times 10^{4}$ eV•s/m$) = 3.2 \times 10^{-19}$ m.
To estimate the diameter of either proton, we first look at the result of our analysis. Here is the histogram from the Kansas State University QuarkNet Center.



Notice that the peak at 160 μrad and the neighboring peak at 220 μrad. The peak spacing is then 60 μrad. Given that the central maximum (n=0) is at 0 μrad and that the region inside the beampipe extends from about -140 μrad to +140 μrad. We can visually estimate the order of the maximum at 160 μrad to be n=2. Assigning the n value is in part a judgement call. The choice of assigning an n value to each peak is another source of uncertainty in our value for the proton radius.

Now we have what we need to find the approximate proton diamter.As noted above, the simple equation for locating maxima in an interference pattern is $\sin \theta_n = n\lambda/d$. To find proton diameter d and recognizing that, for these very small angles, we can say $\sin \theta_n = \theta_n$, we rewrite this as $d = n\lambda/\theta_n$. Thus:

$d = n\lambda/\theta_n = (2)(3.2 \times 10^{-19}\ \text{m})/(160 \times 10^{-6}\ \mu\text{rad}) = 4 \times 10^{-15}\ \text{m}$ = 4 femtometers (fm).

The most recent average of measuements of the "charge radius of the proton" by the Particle Data Group is 0.8409 fm. The diameter is twice this value, or about 1.7 fm. Our result is not the same number but agrees with the order of magnitude.

Discussion questions:

- What are the parts of the model that you used to find your result?
  - *Colliding protons have wavelike properties and follow the rules of optical interference.*
  - *Since protons behave like waves we can determine the proton wavelength using the deBroglie wavelength equation.*
  - *One proton acts like a wave and the other proton acts like a barrier.*
  - *The barrier width represents the diameter of the proton.*
- What evidence supports the model?
  - *Interference – the histogram formed an interference pattern.*
  - *Using the model, the result was consistent with known values of the diameter of a proton to the correct order of magnitude.*
- Describe which assumptions cause our model to fall short.
  - *The measurement tool must fit the need. For better precision in measurement, the precision of the measuring tool must be small compared to the size of the item measured. In this case, the probing particle is the same size as the object undergoing measurement so we would expect a less precise answer. To that end, we are happy with an order of magnitude result.*
  - *We used a classical model of interference for quantum objects.*

ASSESSMENT

The assessment is based on claims, evidence, and reasoning.

One summative approach is to ask the students to complete a report in which they assess the accuracy of each claim and support the claim using evidence from the analysis. It is important to connect the data used as evidence by reasoning back to the claim.

Claims we hope the students will be able to make based on the evidence they build:

- The scattering angles of individual protons, plotted altogether in a histogram, reveal a interference pattern, showing, as de Broglie predicted, that quantum objects such as protons can behave like waves.

- We can find the order of magnitude of the diameter of a proton based on analysis of wave interference.

Formative assessments can occur during the activity by going to each group and asking questions to determine the level of student involvement and understanding.